

# Probabilistic Technique for PAPR Reduction in OFDM System

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**Abstract** -One major drawback of Orthogonal Frequency Division Multiplexing (OFDM) is the large peak-to-average power-ratio (PAPR), which significantly degrades the power efficiency. In order to prevent nonlinear distortions the PAPR needs to be minimized to guarantee a linear dynamic range of the high power amplifier. Therefore, various reduction algorithms have been proposed. This paper investigates two probabilistic techniques, namely, selected mapping (SLM) and partial transmit sequence (PTS) for the purpose of reducing PAPR. From the analysis, it is inferred that PTS method provides a better PAPR reduction performance compared to SLM method. However, the transmitter and receiver complexity is very high. Thus in practical applications, a tradeoff needs to be made between good performance and auxiliary information. It is also found that SLM algorithm is more suitable if system can tolerate more redundant information; otherwise, PTS algorithm is more acceptable when complexity becomes the first considering factor.

**Index Terms** -OFDM, PAPR, CCDF, SLM, PTS.

## I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has come to the forefront of technology over past decade because of its robustness against multipath fading channels. It is an effective high-speed data transmission scheme without using very expensive equalizers and for this reason, forms the basis of the physical layer of many broadband high data rate technologies including Digital Subscriber Line (XDSL), WiFi (IEEE802.11a/g), WiMAX (IEEE802.16a/e), and Digital Video Broadcasting (DVB) [1]. OFDM uses the spectrum much more efficiently by spacing the channels more closely together. One of the major drawbacks of OFDM systems is that the OFDM signal exhibits a high Peak to Average Power Ratio (PAPR). Such a high PAPR necessitates the linear amplifier to have a large dynamic range which is difficult to accommodate. On the other hand, an amplifier with nonlinear characteristics will cause undesired distortion in band and out of band of the signals.

A large number of PAPR reduction techniques have been proposed in the literature. Among them, schemes like phase optimization [2], constellation shaping [3], nonlinear companding transforms [4], tone reservation (TR), tone injection (TI) [5][6], clipping and filtering [7], precoding based techniques [8][9] and phase modulation transform [10] are popular. Among these, two signal scrambling techniques, namely, selected mapping (SLM) and partial transmit sequence (PTS) are investigated in this paper.

## II. OFDM SIGNAL AND PAPR

In this section, we review the basic of OFDM transmitter and the PAPR definition. Consider an OFDM signal consisting of  $N$  subcarriers. Let a block of  $N$  symbol  $X = \{X_k, k = 0, 1, \dots, N-1\}$  is formed with each symbol modulating one of a set of subcarriers  $\{f_k, k = 0, 1, \dots, N-1\}$ . The  $N$  subcarriers are chosen to be orthogonal, that is,  $f_k = k\Delta f$ , where  $\Delta f = 1/(NT)$  and  $T$  is the original symbol period. Therefore, the complex baseband OFDM signal can be written as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi f_k t}, 0 \leq t \leq NT \quad (1)$$

In general, the PAPR of OFDM signals  $x(t)$  is defined as the ratio period between the maximum instantaneous power and its average power during an OFDM symbol

$$PAPR\{x(t)\} = \frac{\max_{0 \leq t \leq T_s} |x(t)|^2}{E\{|x(t)|^2\}} \quad (2)$$

Reducing the  $\max|x(t)|$  is the principle goal of PAPR reduction techniques. In practice, most systems deal with a discrete-time signal, therefore, we have to sample the continuous time signal  $x(t)$ .

To better approximate the PAPR of continuous time OFDM signals, the OFDM signals samples are obtained by  $L$  times oversampling. By sampling defined in (1), at frequency

$f_s = L/T$ , where  $L$  is the oversampling factor, the discrete time OFDM symbol can be written as

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j\frac{2\pi kn}{NL}} \quad (3)$$

Equation (3) can be implemented by using a length  $(NL)$  IFFT operation. The new input vector  $X$  is extended from original  $X$  by using the so-called zero-padding scheme, by inserting  $(L - 1)N$  zeros in the middle of  $X$ . The PAPR computed from the  $L$  - times oversampled time domain OFDM signal samples can be defined as

$$PAPR[x(n)] = 10 \log \frac{\max_{0 \leq t \leq NL-1} |x(n)|^2}{E\{|x(n)|^2\}} \quad (4)$$

However, the PAPR does not increase significantly after  $L = 4$ . In order to avoid aliasing the out of band distortion into the data bearing tones and in order to accurately describe the PAPR an oversampling factor  $L \geq 4$  is required.

We can evaluate the performance of PAPR using the cumulative distribution of PAPR of OFDM signal. The cumulative distribution function (CDF) is one of the most regularly used parameters, which is used to measure the efficiency of and PAPR technique. The CDF of the amplitude of a signal sample is given by

$$F(Z) = 1 - \exp(-Z) \quad (4)$$

However, the complementary CDF (CCDF) is used instead of CDF, which helps us to measure the probability that the PAPR of a certain data block exceeds the given threshold. The CCDF of the PAPR of the data block is desired is our case to compare output of various reduction techniques. This is given by

$$P(PAPR > Z) = 1 - P(PAPR \leq Z) = 1 - F(Z)^N = 1 - (1 - \exp(-Z))^N \quad (5)$$

### III. SELECTIVE MAPPING TECHNIQUE SLM

Fig.1, shows the block diagram of selective mapping (SLM) technique for PAPR reduction. Here, the input data block  $X = [X[0], X[1], \dots, X[N - 1]]$  is multiplied with  $U$  different phase sequences  $P^u = [P_0^u, P_1^u, \dots, P_{N-1}^u]^T$  where  $P_v^u = e^{j\varphi_v^u}$  and  $\varphi_v^u \in [0, 2\pi]$  for  $v = 0, 1, \dots, N - 1$  and  $u = 1, 2, \dots, U$ , which produce a modified data block  $X^u =$

$[X^u[0], X^u[1], \dots, X^u[N - 1]]^T$ . IFFT of  $U$  independent sequences  $\{X^u[v]\}$  are taken to produce the sequences  $x^u = [x^u[0], x^u[1], \dots, x^u[N - 1]]^T$ , among which the one  $\tilde{x} = x^{\tilde{u}}$  with the lowest PAPR is selected for transmission [11], as shown as

$$\tilde{u} = \operatorname{argmin}_{u=1,2,\dots,U} \left( \max_{n=0,1,\dots,N-1} |x^u[n]| \right) \quad (6)$$

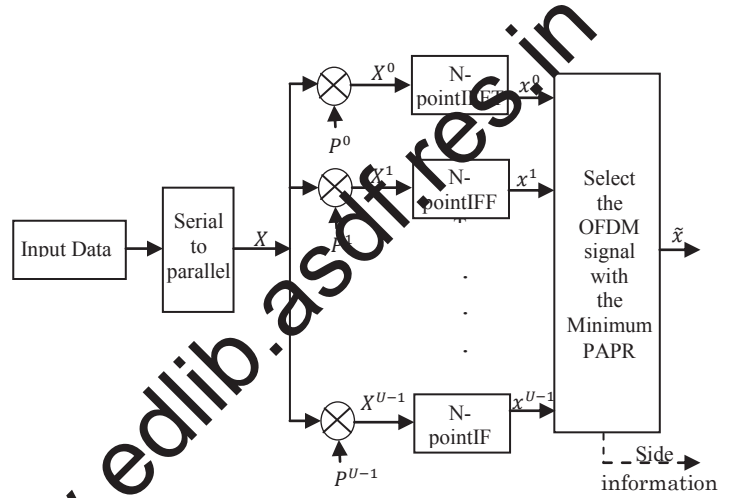


Fig. 1. Block diagram of selective mapping (SLM) technique for PAPR reduction.

In order for the receiver to be able to recover the original data block, the information (index  $u$ ) about the selected phase sequence  $P^u$  should be transmitted as a side information [11]. The implementation of SLM technique requires  $U$  IFFT operations. Furthermore, it requires  $\log_2 U$  bits of side information for each data block.

### IV. PARTIAL TRANSMIT SEQUENCE PTS

The partial transmit sequence (PTS) technique partitions an input data block of  $N$  symbols into  $V$  disjoint sub blocks as follows:

$$X = [X^0, X^1, \dots, X^{V-1}]^T \quad (7)$$

where  $X^i$  are the sub blocks that are consecutively located and also are of equal size. Unlike the SLM technique in which scrambling is applied to all subcarriers, scrambling (rotating its phase independently) is applied to each sub block [11] in the PTS technique. Then each partitioned subblock is multiplied by a corresponding complex phase factor  $b^v = e^{j\theta^v}$ ,  $v = 1, 2, \dots, V$ . Subsequently taking its IFFT to yield

$$x = IFFT \left\{ \sum_{v=1}^V b^v X^v \right\} = \sum_{v=1}^V b^v \cdot IFFT \{X^v\} = \sum_{v=1}^V b^v x^v \quad (8)$$

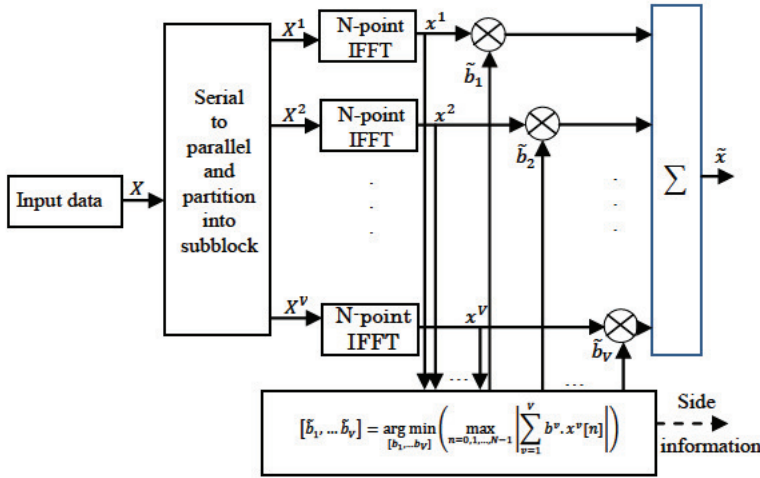


Fig 2. Block diagram of partial time sequence (PTS) technique for PAPR reduction

where  $\{x^v\}$  is referred to as a partial transmit sequence (PTS). The phase vector is chosen so that the PAPR can be minimized [11], which is shown as

$$[\tilde{b}^1, \dots, \tilde{b}^V] = \operatorname{argmin}_{[b^1, \dots, b^V]} \left( \max_{n=0,1,\dots,N-1} \left| \sum_{v=1}^V b^v x^v[n] \right| \right) \quad (9)$$

Then, the corresponding time-domain signal with the lowest PAPR vector can be expressed as

$$\tilde{x} = \sum_{v=1}^V \tilde{b}^v x^v \quad (10)$$

In general, the selection of the phase factors  $\{b^v\}_{v=1}^V$  is limited to a set of elements to reduce the search complexity [12]. As the set of allowed phase factors is  $b = \{e^{j2\pi i/W}, i=0,1,\dots,W-1\}$ ,  $W^V$  sets of phase factors should be searched to find the optimum set of phase vectors. Therefore, the search complexity increases exponentially with the number of subblocks.

The PTS technique requires  $V$  IFFT operations for each data block and  $\log_2 W^V$  bits of side information. The PAPR performance of the PTS technique is affected by not only the number of sub blocks,  $V$ , and the number of the allowed phase factors,  $W$ , but also the sub block partitioning.

## V. SIMULATION RESULTS

### A. Selective Mapping Technique

In this part, an evaluation of factors which could influence the PAPR reduction performance is performed using Mat lab simulation. Based on the principles of SLM algorithm, it is apparent that the ability of PAPR reduction using SLM is affected by the vector number  $U$  and subcarrier number  $N$ .

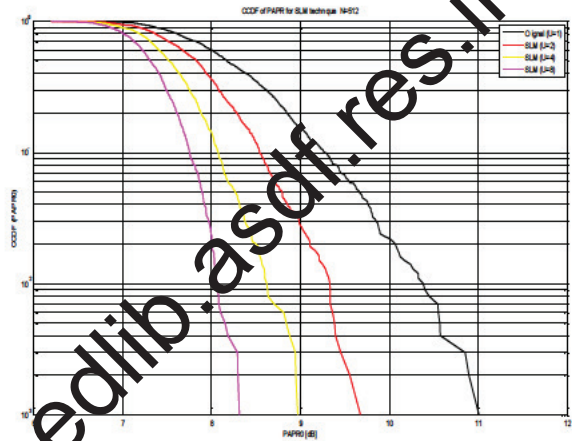


Fig 3. Performance of SLM PAPR reduction scheme with different values of  $U$ .

Fig 3.shows that the vector numbers  $U=2$ ,  $U=4$  and  $U=8$  are used, while  $N$  is fixed at 512. We have observed that the proposed SLM method displays a better PAPR reduction performance than the original OFDM signal indicates that when the vector numbers increase the PAPR decrease. However, it is difficult to achieve a linear growth of PAPR reduction performance with further increase in the value of  $U$  (like  $U > 8$ ). Therefore in practical applications, it is preferred to take  $U=8$ , so as to avoid introducing too much computational complexity.

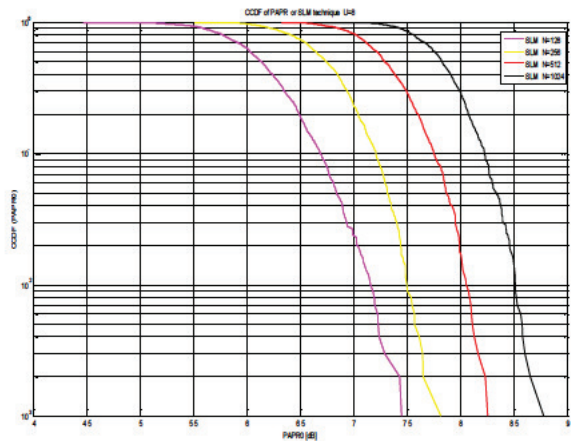


Fig 4. Performance of SLM PAPR reduction scheme with different values of  $N$ .

Fig 4, shows that the set number of OFDM signal frame  $U$  equals to 8, the number of sub-carrier  $N$  equals to 1024, 512, 256 and 128 respectively. It can be seen that when the number of sub-carrier increase the PAPR increase. In terms of complexity, every time when SLM algorithm is applied, it requires calculating the  $U$  group IFFTs at the transmitter compared to only one on ordinary OFDM system; therefore, it is required to reduce the computational complexity. In practical applications, to compromise with the computing complexity and improve the performance,  $U \leq 8$  is usually taken.

*B. Partial Transmit Sequence Technique*

The PAPR reduction performance in PTS approach is affected by the number of sub-blocks  $V$ , which influences the complexity strongly; the number of possible phase value  $W$ , which impacts the complexity as well and the number of sub-carrier  $N$ .

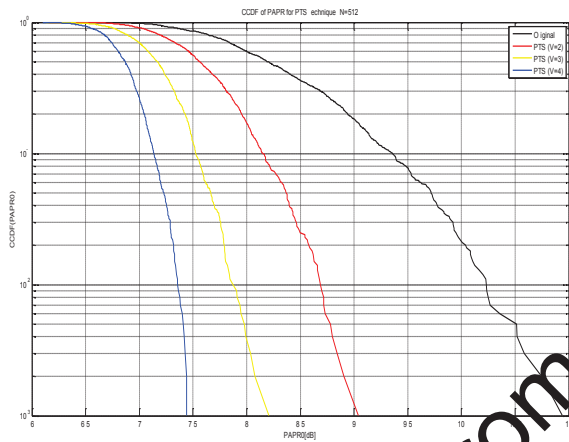


Fig 5. Performance of PTS PAPR reduction scheme with different values of  $V$ .

Fig 5, shows the CCDF of PAPR for a QPSK/OFDM system using PTS Technique as the number of sub-block varies. It is seen that the PAPR performance improves as the number of sub-blocks increases with  $V = 1, 2, 4$  and  $8$ . So by increasing the number of sub-blocks, the improvement of PAPR reduction performance becomes better and better. It can also be observed that with increasing the value of  $V$ , the PAPR is more reduced. However for higher values of  $V$ , the CCDF curve's tail off is less with increase in the value of  $V$ . This means large sub-block numbers  $V$  will result in small improvement of PAPR reduction performance and increases the hardware complexity. Therefore in practice, it is advisable to choose a suitable value of  $V$  to achieve a tradeoff in the use of PTS.

Fig 6, shows that the set number of OFDM signal subblock  $V=8$ , the number of sub-carrier  $N$  equals to 1024, 512, 256 and 128 respectively. It can be seen that when the number of sub-carrier increase the PAPR increase. In terms of

complexity, every time when PTS algorithm is applied, it requires calculating the  $V \cdot W$  group IFFTs at the transmitter compared to only one on ordinary OFDM system; therefore, it is required to reduce the computational complexity.

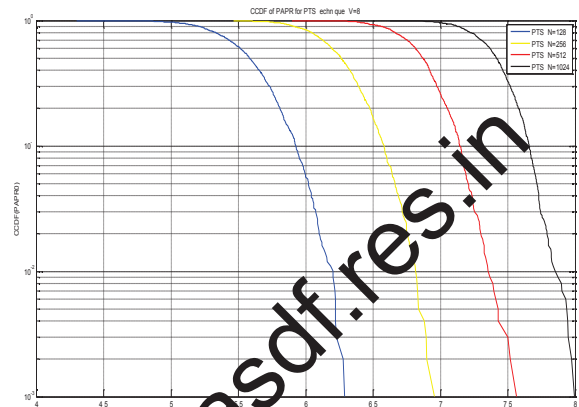


Fig 6. Performance of PTS PAPR reduction scheme with different values of  $N$ .

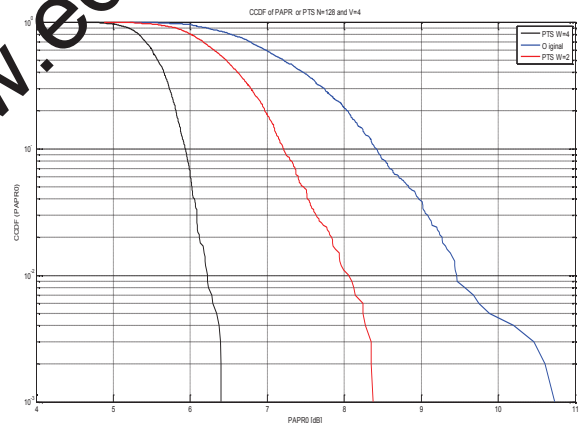


Fig 7. Performance of PTS PAPR reduction scheme with different values of  $W$ .

Fig 7, shows the PAPR performance of OFDM system with PTS technique with different number phases value  $W$ . We conclude that in a PTS-OFDM system, the larger  $W$  value takes, the better PAPR performance will be obtained when the number of sub-block  $V$  is fixed performance will be obtained when the number of sub-block  $V$  is fixed.

Fig 8, shows a Comparison of PAPR reduction performances between PTS algorithm and SLM algorithm for OFDM system. The PTS method should provide a superior performance on PAPR reduction. In fact this deduction is confirmed by simulation result.

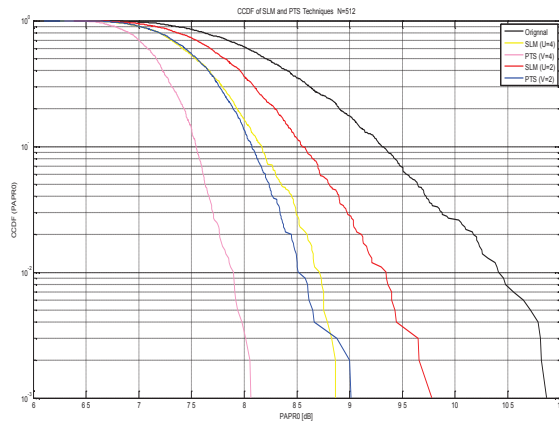


Fig 8. Comparison of PAPR reduction performances between PTS algorithm and SLM algorithm for OFDM systems.

#### IV. CONCLUSION

This paper investigates one of the major drawbacks of OFDM system. High Peak-Average Power Ratio (PAPR of OFDM signal), and discusses how to reduce it by different technique. We are mainly focusing on the probabilistic technique, and discuss it by observing the MATLAB simulation results. In the probabilistic technique, we study the method of selected mapping SLM and partial transmit sequence PTS. A series of detailed simulations were conducted for comparison and results were obtained of the two schemes for PAPR reduction in OFDM system. It has been observed from the results that PTS method provides a better PAPR reduction performance compared to SLM method. However, the transmitter and receiver complexity is very high. Thus in practical applications, a tradeoff needs to be made between good performance and auxiliary information. From the above made discussion SLM algorithm is more suitable if system can tolerate more redundant information; otherwise, PTS algorithm is more acceptable when complexity becomes the first considering factor.

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