Control of chaotic satellites systems based on the predictive control and passive control

A. Melit, K. Kemih
Faculty of Sciences and Technology, L2EI Laboratory, Jijel University, 18000, Algeria

***Abstract*- In this paper, the problem of attitude stabilization was discussed by using the predictive control and the passive control theory. These controls are applied to the attitude equation of satellite which includes some types of nonlinear behavior such as periodic trajectory, chaotic dynamics. Moreover, the conditions under which the chaotic systems can be asymptotically controlled to the origin are presented. Simulation results are provided to show the effectiveness of the proposed control methods**

Keywords: Chaotic satellite attitude, Predictive control, passive control.

I. INTRODUCTION

In recent years, chaos theory has drawn great attention because of its theoretical importance and application in attitude dynamics of spacecraft. Chaotic systems are characterized by being extremely sensitive to initial conditions, deterministically random, and hence ultimately unpredictable.

In order to control the chaotic evolution of these systems, many nonlinear control methods have been developed. Feedback linearization technique to spacecraft attitude control has been discussed in [1-2]. Impulsive control has been used in [3-6], Generalized Predictive Control approach has been used in [7], and a sliding-mode approach has been used in [8]. Lyapunov-based control technique has been utilized in [9] and LMI-based nonlinear control in [10]. Nonlinear H∞ control in [11].

In this paper, we investigate the stability of chaotic satellite attitude control (angular velocities and Attitude angles) using the feedback predictive control and the passive control methods. Some new sufficient conditions are derived for stabilizing the chaotic system. Finally, a comparison between the results obtained by the two proposed control is provided, it is based on the same initial conditions and the same moment of inertia.

II. STABILISATION OF SATELLITE

In this paper, the passive and the feedback predictive control are used to stabilize the satellite attitude given by the equations (1) and (2).

Satellite attitude kinematic equation is as follower:

$$\begin{aligned}\dot{\phi} &= w_x + \sin\phi\tan\theta w_y + \cos\phi\tan\theta w\\ \dot{\theta} &= \cos\phi w_y - \sin\phi w_z\\ \dot{\psi} &= \sin\phi\sec\theta w_y + \cos\phi\sec\theta w_z\end{aligned} \tag{1}$$

The angular velocities are determined by a system of first order differential equations (Euler equations):

$$\begin{cases}\mathrm{I_X}\dot{w}_\mathrm{X} = (\mathrm{I_Y}-\mathrm{I_Z})w_\mathrm{Y}w_\mathrm{Z} + C_\mathrm{X}\\ \mathrm{I_Y}\dot{w}_\mathrm{Y} = (\mathrm{I_Z}-\mathrm{I_X})w_\mathrm{X}w_\mathrm{Z} + C_\mathrm{Y}\\ \mathrm{I_Z}\dot{w}_\mathrm{Z} = (\mathrm{I_X}-\mathrm{I_Y})w_\mathrm{X}w_\mathrm{Y} + C_\mathrm{Z}\end{cases} \tag{2}$$

where Ix, Iy, Iz are the principal moments of inertia w_x, w_y, w_z are angular velocities about the principal x,y,z axes fixed in the rigid body, and Cx, Cy, Cz are torques applied about these axes at time t.

Earlier papers [14-16] have taken Ix= 3000 kg.m2, Iy = 2000 kg.m2 and Iz = 1000 kg.m2 with the perturbing torques defined by:

$$\begin{bmatrix}C_x\\C_y\\C_z\end{bmatrix} = \begin{bmatrix}-1200 & 0 & 1000\sqrt{6}/2\\ 0 & 350 & 0\\ -1000\sqrt{6} & 0 & -400\end{bmatrix}\begin{bmatrix}\omega_x\\\omega_y\\\omega_z\end{bmatrix} + \begin{bmatrix}1000\times\cos\theta\sin\psi\\ 1000\times\cos\phi\sin\theta\\ 1000\times\cos\psi\sin\theta\end{bmatrix} \tag{3}$$

These torques are chosen to be sufficiently large to induce very high chaotic motion and are comparable in magnitude with the available thrusters torques. The dynamics of the satellite will then exhibit chaotic motion [15].

The temporal evolution of the system defined by (1) and (2) is shown in Figures. 1 and 2.

A. Stabilisation using the passive control

Passivity is applied to nonlinear systems which are modeled by ordinary differential equations with input vector $u(t)$ and output vector $y(t)$ [13]:

$$\begin{aligned}\dot{x}(t) &= f(x(t), u(t))\\ y(t) &= h(x(t)\end{aligned} \tag{4}$$

The system (4) is dissipative with the supply rate $W(u(t), y(t))$, if it is not able to generate power by itself, that is, the energy stored in the system is less than or equal to the supplied power:

$$V(x(t)) \geq 0, \quad V(x(T)) - V(x(0)) \leq \int_0^T W(u(t), y(t))\,dt \qquad (5)$$

Furthermore, the storage function $V(x(t))$ must satisfy the requirements for a Lyapunov function.

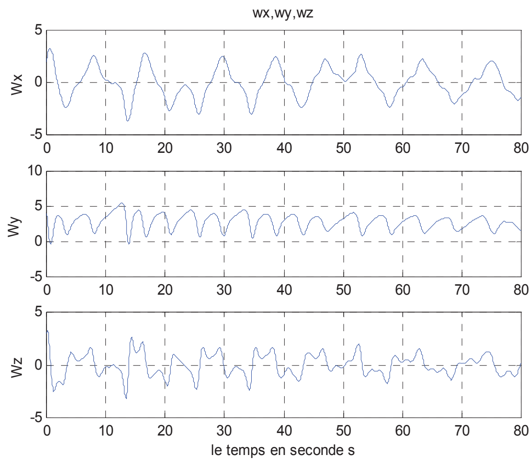

Figure 1. Angular velocities.

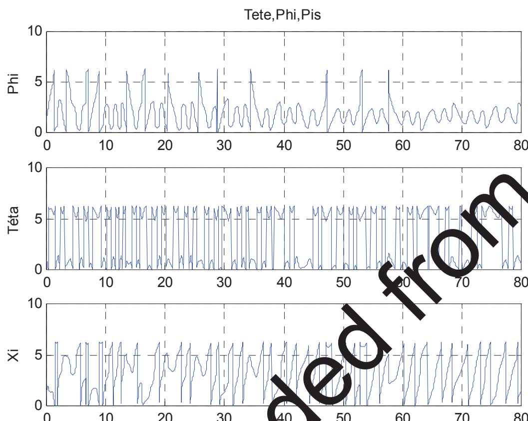

Figure 2. Attitude angles.

If there exists a positive semi definite Lyapunov function, such that:

$$\int u^T(t)y(t)dt \geq \int [\frac{\partial V(x(t))}{\partial x(t)} f(x(t),u(t)) + eu^T + \delta y^T(t)y(t) + \rho\phi(x(t))]dt \qquad (6)$$

then the system (4) is passive. A passive system implies that any increase in storage energy is due solely to an external power supply.

Then the equilibrium point of the system: $\dot{x}(t) = f(t, x(t), 0)$ is asymptotically stable in either of the two cases:

(i) $\rho > 0$

(ii) $e + \delta > 0$ (7)

The system (4) can be represented as the normal form [17]:

$$\dot{z} = f(z) + g(z,y)y$$
$$\dot{y} = l(z,y) + k(z,y)u \qquad (8)$$

The nonlinear system (8) may be rendered by a state feedback as the form:

$$u = \alpha(z) + \beta(z)y \qquad (9)$$

We consider the system (1) and (2) as follows:

$$\begin{cases} \dot\phi = w_x + \sin\phi\tan\theta w_y + \cos\phi\tan\theta w_Z \\ \dot\theta = \cos\phi w_y - \sin\phi w_Z \\ \dot\psi = \sin\phi\sec\theta w_y + \cos\phi\sec\theta w_z \\ \dot w_X = \frac{(I_Y - I_Z)}{I_X} w_Y w_Z - \frac{1.2}{I_X} w_X + \frac{\sqrt{6}}{2I_X} w_Z + \frac{\cos\theta\sin\psi}{I_X} + \frac{T_x}{I_x} \\ \dot w_Y = \frac{(I_Z - I_X)}{I_Y} w_X w_Z + \frac{0.35}{I_Y} w_Y + \frac{\cos\phi\sin\theta}{I_Y} + \frac{T_y}{I_y} \\ \dot w_Z = \frac{(I_X - I_Y)}{I_Z} w_X w_Y - \frac{\sqrt{6}}{I_Z} w_X - \frac{0.4}{I_Z} w_Z + \frac{\cos\psi\sin\phi}{I_Z} + \frac{T_z}{I_z} \end{cases} \qquad (10)$$

Suppose that state variable w_y is the output of the system and suppose $w_x = z_1, w_y = y, \; w_z = z_2, \; \phi = z_3, \; \theta = z_4$ and $\psi = z_5$ then the system (10) can be expressed by passive form (8) where:

$$f(z) = \begin{bmatrix} \frac{1}{I_z}(-\sqrt{6}z_1 - 0.4z_2 + \cos z_5 \sin z_4) \\ z_1 + \cos z_3 \tan z_4 z_2 \\ -\sin z_3 z_2 \\ \cos z_3 \sec z_4 z_2 \end{bmatrix}, \; g(z,y) = \begin{bmatrix} \sigma_x z_2 \\ \sigma_z z_1 \\ \sin z_3 \tan z_4 \\ \cos z_3 \\ \sin z_3 \sec z_4 \end{bmatrix}$$

$$l(z,y) = \sigma_y z_1 z_2 + \frac{1}{I_y}(0.35y + \cos z_3 \sin z_4), \; k(z,y) = 1 \qquad (11)$$

Our object is to design a smooth control (9) to make the closed-loop system passive.

Choose a storage function candidate:

$$V(z,y) = W(z) + \frac{1}{2}y^2 \qquad (12)$$

where $W(z)$ is Lyapunov function, with $W(0) = 0$.

The Lyapunov function is:

$$W(z) = \frac{1}{2}(z_1^2 + z_2^2 + z_3^2 + z_4^2 + z_5^2) \qquad (13)$$

Derivative of the Lyapunov function is as follows:

$$\frac{d}{dt}W(z) = z_1(-0.4z_1 + \frac{\sqrt{6}}{6}z_2 + \frac{1}{3}\cos z_4 \sin z_5) + z_2(-\frac{\sqrt{6}}{I_z}z_1 - \frac{0.4}{I_z} + \cos z_5 \sin z_4) + z_3(z_1 + \cos z_3 \tan z_4 z_2) - z_4(\sin z_3 z_2) + z_5(\cos z_3 \sec z_4 z_2) \leq 0 \qquad (14)$$

 ISBN : 978-81-925233-8-5

From equation (14), the zero dynamics of the chaotic system is asymptotically stable in the sense of Lyapunov.

The derivative of $V(z, y)$ along the trajectory of system (10) is

$$\frac{d}{dt}V(z,y)=\frac{\partial}{\partial z}W(z)f(z)+\frac{\partial}{\partial z}W(z)g(z,y)y+l(z,y)y+k(z,y)yu \tag{15}$$

The system is minimum phase:

$$\frac{d}{dt}W(z)f(z)\le 0 \tag{16}$$

Equation (15) becomes:

$$\frac{d}{dt}V(z,y)\le\frac{\partial}{\partial z}W(z)g(z,y)y+(l(z,y)+k(z,y)u)y \tag{17}$$

If we select the feedback control (9) of the following form and consider (11):

$$\begin{aligned}u&=k^{-1}(z,y)[-l^T(z,y)-\frac{\partial W}{\partial z}g(z,y)-\gamma y+\upsilon]\\&=-(\sigma_x+\sigma_y+\sigma_z)z_1z_2-\frac{0.35}{\mathrm{I}_y}y-\gamma y-\frac{1}{\mathrm{I}_y}\cos z_3\sin z_4\\&\quad-z_3\sin z_3\tan z_4-z_4\cos z_3-z_5\sin z_3\sec z_4+\upsilon\end{aligned} \tag{18}$$

where γ is positive constant, and υ is an external signal which is connected with the reference input, the above inequality can be rewritten as

$$\frac{d}{dt}V(z,y)\le-\gamma y^2+\upsilon y \tag{19}$$

Then by integrating both sides of (19), we have

$$V(z,y)-V(z_0,y_0)\le\int_0^t-\gamma y^2(\tau)d\tau+\int_0^t\upsilon(\tau)y(\tau)d\tau \tag{20}$$

If $V(z,y)\ge 0$ and $\rho=V(z_0,y_0)$ then

$$\int_0^t\upsilon(\tau)y(\tau)d\tau+\rho\ge V(z,y)+\int_0^t\gamma y^2(\tau)d\tau\ge\int_0^t\gamma y^2(\tau)d\tau \tag{21}$$

It satisfies the passive definition (6), the system (10) is rendered to be output strict passive (OSP) under the feedback control.

The system is simulated by the fourth-order Runge–Kutta integration method with the following initial conditions (wx, wy, wz, Φ, θ, ψ) = (1,1,1,1,1,1) and use the controller as in (18).

The simulation results are shown in Figs. 3 and 4 for different values of γ. From the figures we have that the state variables quickly tend to the origin under passive control, also the bigger γ gives the best performance.

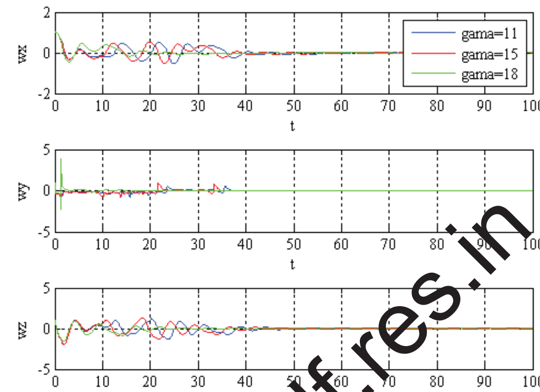

Figure 3. Controlled angular velocities.

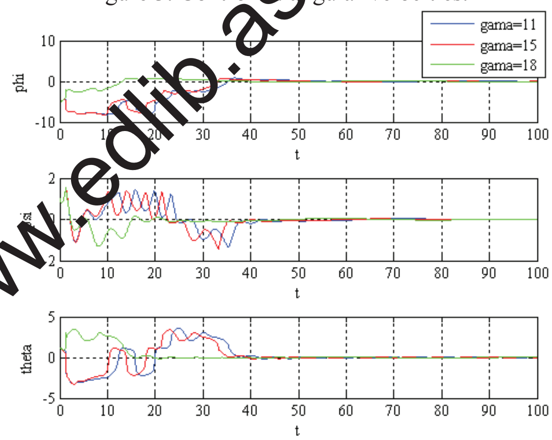

Figure 4. Controlled attitude angles.

B. *Stabilisation using the predictive control*

Consider the nonlinear system described by [18-19]

$$\dot{x}(t)=f(x(t),u(t)) \tag{22}$$

where $x\in R^n$ is the state vector and $u\in R^n$ is the feedback controller. We assume that f is differentiable.

The control input u(t) is determined by the difference between the predicted states and the current states:

$$u(t)=K(x_p(t)-x(t)) \tag{23}$$

where K is a gain vector, $x_p(t)$ is the predicted future state of uncontrolled chaotic systems from the current state $x(t)$.

Using a one-step-ahead-prediction, the predictive control (23) becomes

$$u(t)=K(\dot{x}(t)-x(t)) \tag{24}$$

Near x_f, we can use the linear approximation for the uncontrolled system by

$$\dot{x}(t) - x_f = A(x(t) - x_f) \tag{25}$$

where $A \in R^{n\times n}$ is the Jacobian which is defined as follows:

$$A = D_x f(x_f) = \left.\frac{\partial \dot{x}(t)}{\partial x(t)}\right|_{X_f} \tag{26}$$

The controlled system will be described by:

$$\dot{x}(t) = f(x(t)) + K(A(x(t) - x_f)) \tag{27}$$

Eq. (25) is rewritten in the form

$$\partial\dot{x}(t) = A\partial x(t) \tag{28}$$

With

$$\partial x(t) = x(t) - x_f$$

(29)

The controlled system is linearized around x_f by

$$\begin{aligned}\delta\dot{x}(t) &= A\delta x(t) + K(\delta\dot{x}(t) - \delta x(t)) \\ &= A\delta x(t) + K(A\delta x(t) - \delta x(t)) \\ &= (A + K(A - \mathrm{I})\delta x(t)\end{aligned} \tag{30}$$

The feedback gain K is determined as follows [18]:

$$|A + K(A - \mathrm{I})| < \mathrm{I} \tag{31}$$

And the vicinity of the fixed point is given by:

$$r(t) = |x(t) - x(t-1)| \tag{32}$$

The controlled system will be described by [19]:

$$\dot{x}(t) = \begin{cases} f(x(t)) + u(t) & \text{if } r(t) < \varepsilon \\ f(x(t)) & \text{otherwise,} \end{cases} \tag{33}$$

Where ε is a positive small real number.

We consider the system as follows:

$$\begin{cases} \dot{w}_X = \dfrac{(I_Y - I_Z)}{I_X} w_Y w_Z - \dfrac{1.2}{I_X} w_X + \dfrac{\sqrt{6}}{2I_X} w_Z \\ \dot{w}_Y = \dfrac{(I_Z - I_X)}{I_Y} w_X w_Z + \dfrac{0.35}{I_Y} w_Y \\ \dot{w}_Z = \dfrac{(I_X - I_Y)}{I_Z} w_X w_Y - \dfrac{\sqrt{6}}{I_Z} w_X - \dfrac{0.4}{I_Z} w_z \end{cases} \tag{34}$$

In order to control the system to the unstable equilibrium point $[0\,0\,0]^T$, we have to determine the correction which will be applied to the current state of the chaotic system. For this purpose, we determine the control input u(t) defined by Eq.(24).

$$u(t) = \begin{cases} 0 \\ K((\sigma_y w_x w_z + \dfrac{0.35}{I_y} w_y - w_y) \\ 0 \end{cases} \tag{35}$$

The controlled system is given by:

$$\begin{cases} \dot{w}_X = \dfrac{(I_Y - I_Z)}{I_X} w_Y w_Z - \dfrac{1.2}{I_X} w_X + \dfrac{\sqrt{6}}{2I_X} w_Z \\ \dot{w}_Y = \dfrac{(I_Z - I_X)}{I_Y} w_X w_Z + \dfrac{0.35}{I_Y} w_Y + K((\dfrac{I_z - I_x}{I_y}) w_x w_z + \dfrac{0.35}{I_y} w_y - w_y) \\ \dot{w}_Z = \dfrac{(I_X - I_Y)}{I_Z} w_X w_Y - \dfrac{\sqrt{6}}{I_Z} w_X - \dfrac{0.4}{I_Z} w_z \end{cases} \tag{36}$$

Its linearized system around the fixed point is given by:

$$\begin{aligned}\delta\dot{w}_y(t) &= \frac{\delta\dot{w}_y(t)}{\delta w_y(t)}\delta w_y(t) \\ &= (\frac{0.35}{I_y} + K(\frac{0.35}{I_y} - 1))\delta w_y(t)\end{aligned} \tag{37}$$

Then, K must satisfy the inequality:

$$\left|\frac{0.35}{I_y} + K(\frac{0.35}{I_y} - 1)\right| < 1 \tag{38}$$

This implies that:

$$-1 < K < 1.424 \tag{39}$$

And the vicinity of the fixed point is given by:

$$r(t) = |w_y(t) - w_y(t-1)| \tag{40}$$

Thus, the controlled system is described by:

$$u(t) = \begin{cases} K((\sigma_y w_x w_z + \dfrac{0.35}{I_y} w_y - e_2) & si\,|w_y(t) - w_y(t-1)| < \varepsilon \\ 0 & ailleurs \end{cases} \tag{41}$$

In the simulation process the initial states of system are $(w_x,w_y,w_z)=(1, 1, 1)$ and the gain vector is K = [0 0.5 0]. The simulation results are shown in Figs. 5 and 6 for k=0.5 and K=3.

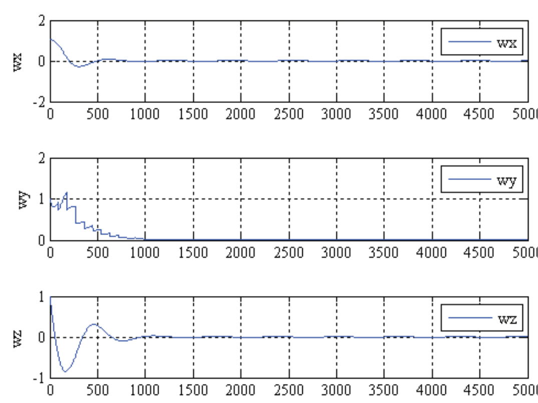

Figure 5. Controlled angular velocities for k=0.5.

 ISBN : 978-81-925233-8-5

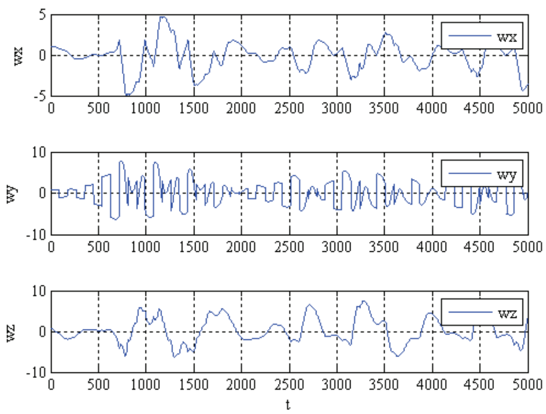

Figure 6. Controlled angular velocities for k=3.

From the simulation results, it shows that the time responses of the system under the proposed feedback predictive control converge quickly to zero if the values of K are in the interval $[0,1.424]$. And if the value of K is out of the interval, the predictively controlled system cannot be stabilized

The following figures show the variations of the angular velocities controlled by the feedback predictive control, passive control and the impulsive method based on the same initial conditions.

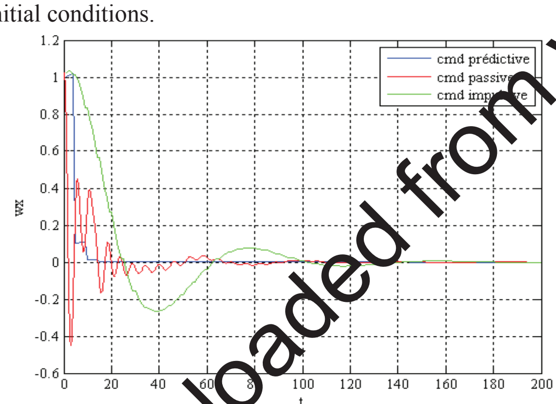

Figure 7. Controlled w_x.

From figures (7), (8) and (9), the results for the passive stabilization were more satisfactory than those for feedback predictive control.

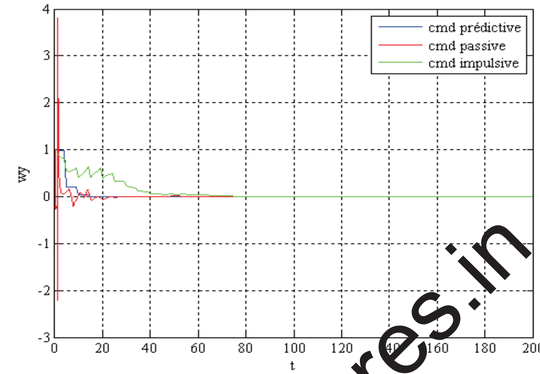

Figure 8. Controlled w_y.

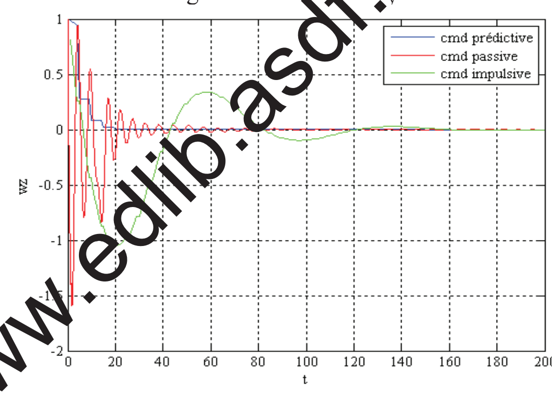

Figure 9. Controlled w_z.

III. CONCLUSION

In this paper, a feedback predictive control and a passive control have been proposed to stabilize the attitude of satellite.

Necessary and sufficient conditions for stabilization are given. An illustrative example is finally included to visualize the effectiveness of each control. At the end a comparison between the simulation results is presented.

REFERENCES

[1] J.J. Sheen and R.H. Bishop, "Spacecraft nonlinear control," J. Astronaut. Sci. vol. 42, no. 3, pp. 361-377, 1994.

[2] C. Li; Guangfu Ma; Bin Song, "Passivity-based Nonlinear Attitude Regulation of Rigid Spacecraft Subject to Control Saturation, WCICA 2006. The Sixth World Congress on Intelligent Control and Automation, vol 2, 21-23 June 2006 Page(s):8421 – 8425.

[3] Dong Li, Shilong Wang, Xiaohong Zhang, Dan Yang, Impulsive control of uncertain Lotka–Volterra predator–prey system, Chaos, Solitons & Fractals, vol 41, no.4, pp 1572–157, 2009

[4] Yinping Zhang, Jitao Sun, Robust synchronization of coupled delayed neural networks under general impulsive control, Chaos, Solitons & Fractals, In Press, Corrected Proof, Available online 22 July 2008

[5] Xiaohong Zhang, Anmar Khadra, Dong Li, Dan Yang Impulsive stability of chaotic systems represented by T-S model, Chaos, Solitons & Fractals, In Press, Corrected Proof, Available online 14 September 2008

[6] Rong Zhang, Zhenyuan Xu, Simon X. Yang, Xueming He, Generalized synchronization via impulsive control, Chaos, Solitons & Fractals, Volume 38, Issue 1, October 2008, Pages 97-105.

[7] K. Kemih, W.-Y. Liu, "Constrained Generalized Predictive Control of Chaotic Lu System", ICIC Express Letters (An International Journal of Research and Surveys), Vol.1, N°1 pp39-44, 2007

[8] J.L. Crassidis and F.L. Markley, "Sliding mode control using modified Rodrigues parameters," J. Guidance Control Dyn. vol. 19, no. 6, pp. 1381-1383, 1996.

[9] D. Fragopoulos, M. Innocenti, "Stability considerations in quaternion attitude control using discontinuous Lyapunov functions," IEE Proc. Control Theory Appl., vol. 151, no. 3, pp. 253-258, 2004.

[10] L.-L. Show, J.-C. Juang, and Y.-W. Jan, "An LMI-based nonlinear attitude control approach," IEEE Trans. Control Syst. Tech., vol. 11, no. 1, pp. 73-83, 2003.

[11] L.-L. Show, J.-C. Juang, Y.-W. Jan, and C.-T. Lin, "Quaternion feedback attitude control design: a nonlinear H∞ approach," Asian J. Control, vol. 5, no. 3, pp. 406-411, 2003.

[12] Manuel Iñarrea, Chaos and its control in the pitch motion of an asymmetric magnetic spacecraft in polar elliptic orbit, Chaos, Solitons & Fractals, In Press, Corrected Proof, Available online 24 October 2007.

[13] M. Nasirian, H. Bolandi, S. A. Kh. Sedigh AR. Khoogar, Design of a satellite attitude control simulator. 1st International Symposium on Systems and Control in Aerospace and Astronautics, 2006. ISSCAA 2006

[14] D.C. Dracopoulos, A.J. Jones, Adaptive neuro-genetic control of chaos applied to the attitude control problem, Neural Comp. Appl. 6 (2) (1997) 102–115.

[15] N Koncar, AJ Jones, Adaptive real-time neural network attitude control of chaotic satellite motion, in: SPIE International Symposium Applications and Science of Artificial Neural Networks, vol. 2492, 1995, pp. 112–123, ISBN 0-8194-1845-5.

[16] A.P.M. Tsui, A.J. Jones, The control of higher dimensional chaos: comparative results for the chaotic satellite attitude control problem, Physica D, V.135, 2000, 41-62.

[17] Wen, Y, "Passive equivalence of chaos in Lorenz system. IEEE Transaction on Circuits and Systems-I": Fundamental Theory and Applications, volume 46, n 7, pp. 876-878, 1999.

[18] Boukabou, A., Chebbah, A., & Mansouri, N. (2008), "Predictive control of continuous chaotic systems", International Journal of Bifurcation and Chaos, volume 18, n 2, pp. 587-592.

[19] Boukabou A. & Mansouri N. "Predictive control of high dimensional chaos,"Nonlinear Phenomena in Complex Systems 8, pp. 258—265, 2005.