Backstepping control of Chaotic Attitude Control of Satellite

K. Kemih*, M. Halimi, H. Fanit*, H. Salit ***

*L2EI Laboratory, Jijel University, Jijel 18000, Algeria

**CRAN/ENSEM, 2 Avenue de la Foret de Haye 54516 Vandœuvre Cedex

Abstract: A backstepping control system is proposed to control the attitude dynamics of a satellite subjected to deterministic external perturbations which induce chaotic motion when no control is affected in this paper. The proposed method is a systematic recursive design approach based on the choice of Lyapunov functions for constructing feedback control laws. The effectiveness of the proposed control scheme is verified by the simulated results.

Keywords: backstepping control, satellite attitude control, chaotic systems, Lyapunov function.

1. INTRODUCTION

Chaotic systems are described by a set of nonlinear and deterministic dynamical equations. Although its equations completely define their evolution, they are unpredictable in the long term. This non-predictability in the long term due to the fact that chaotic systems are very sensitive to initial conditions.

The control of chaotic system has received increased research attention [1-7], since the classical work on chaos control was first presented by Ott and al. [8]. In the last decade, several works interested to attitude control systems of satellites using new advanced nonlinear control theory which ensured better performances. In [9] impulsive control has been used to Chaotic attitude control of satellite. More recently, Mohammad bagheri and all [10] proposed the model predictive control method to stabilize the Lorenz-type chaotic attitude of a satellite.

In other development, Backstepping design has been widely used for controlling chaotic systems [11-13] since backstepping approach provides a recursive method ensure global stability, tracking and transient performance for a board class of system in strict-feedback form. Backstepping approach has been used in [10] to control intermittent chaotic transport in inertia ratchet that model the motion of a particle in an asymmetric periodic potential, and in [11] to the control and synchronization of chaos in RCL-Shunted Josephson junction.

The work presented in this paper deals with the application of the backstepping control system to control the attitude dynamics of a satellite subjected to deterministic external perturbations which induce chaotic motion.

This paper is organized as follows. After this introduction, Sec. 2 focuses on the description of the attitude dynamics of satellite. The matter discussed in Sec. 3 concerns the Backstepping control of Chaotic Attitude Control of Satellite. Finally, simulation results are presented in Sec. 4 in order to shown method effectiveness.

2. DESCRIPTION OF THE ATITTUDE DYNAMICS OF SATELLITE

The dynamical equation of the rigid satellite attitude control system is [14]:

$$\begin{cases} \mathrm{I_X}\,\dot{w}_\mathrm{X} = (\mathrm{I_Y}-\mathrm{I_Z})w_\mathrm{Y}w_\mathrm{Z} + C_\mathrm{X} \\ \mathrm{I_Y}\,\dot{w}_\mathrm{Y} = (\mathrm{I_Z}-\mathrm{I_X})w_\mathrm{X}w_\mathrm{Z} + C_\mathrm{Y} \\ \mathrm{I_Z}\,\dot{w}_\mathrm{Z} = (\mathrm{I_X}-\mathrm{I_Y})w_\mathrm{X}w_\mathrm{Y} + C_\mathrm{Z} \end{cases} \tag{1}$$

where I_X, I_Y, I_Z are the principal moments of inertia, w_x, w_y, w_z are angular velocities about the principal *x, y, z* axes fixed in the rigid body, and C_x, C_y, C_z *are* torques applied about these axes at time *t*. If we choose $I_X = 3000\ kg.m^2$, $I_Y = 2000\ kg.m^2$, and $I_Z = 2000\ kg.m^2$ with the perturbing torques defined by:

$$\begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix} = \begin{bmatrix} -1.2 & 0 & \sqrt{6}/2 \\ 0 & 0.35 & 0 \\ -\sqrt{6} & 0 & -0.4 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{2}$$

The dynamics of the satellite will then exhibit chaotic motion. The chaotic trajectory of the satellite is represented in Figure 1.

3. BACKSTEPPING CONTROL OF CHAOTIC ATTITUDE CONTROL OF SATELLITE WITH ONLY ONE CONTROLLER

Our objective is to stabilize the system (1) to the desired values (w_{1d}, w_{2d}, w_{3d}

).

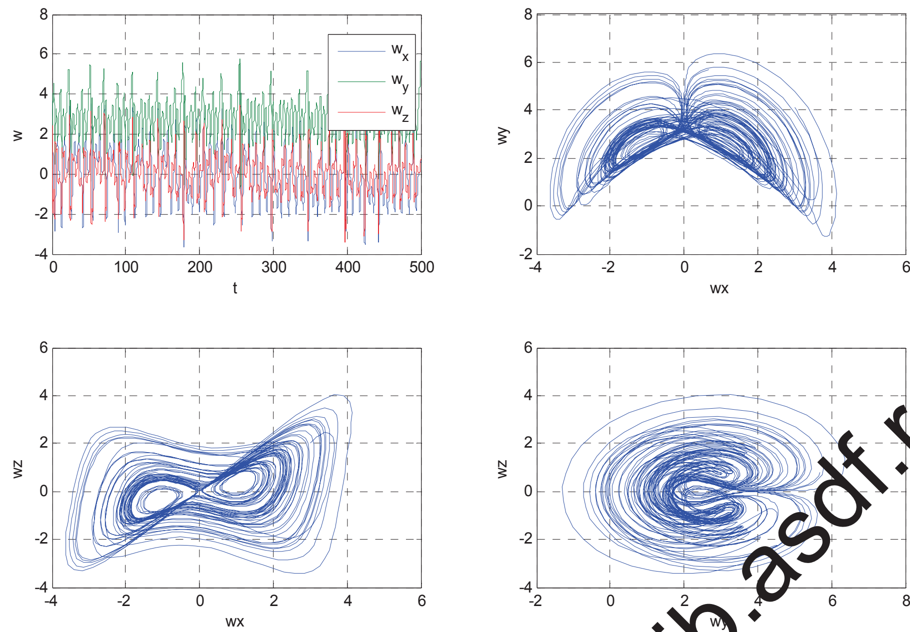

Fig. 1. Chaotic attractor: phase portrait of the angular velocities

Step 1:

$$\begin{cases} \xi_1 = w_1 - w_{1d} \\ \xi_2 = w_2 - w_{2d} \\ \xi_3 = w_3 - w_{3d} \end{cases} \tag{3}$$

Its time derivative is expressed as:

$$\begin{cases} \dot\xi_1 = \dot w_1 - \dot w_{1d} \\ \dot\xi_2 = \dot w_2 - \dot w_{2d} \\ \dot\xi_3 = \dot w_3 - \dot w_{3d} \end{cases} \tag{4}$$

Rewriting equation (4) implies that:

$$\begin{cases} \dot w_1 = \dot\xi_1 + \dot w_{1d} \\ \dot w_2 = \dot\xi_2 + \dot w_{2d} \\ \dot w_3 = \dot\xi_3 + \dot w_{3d} \end{cases} \tag{5}$$

The system (1) becomes:

$$\begin{cases} \dot\xi_1 + \dot w_{1d} = \frac{1}{2}(\xi_2 + w_{2d})(\xi_3 + w_{3d}) - 0.4(\xi_1 + w_{1d}) \\ \qquad -\frac{\sqrt{6}}{6}(\xi_3 + w_{3d}) \\ \dot\xi_2 + \dot w_{2d} = -(\xi_1 + w_{1d})(\xi_3 + w_{3d}) + \frac{0.35}{2}(\xi_2 + w_{2d}) \\ \qquad +u \\ \dot\xi_3 + \dot w_{3d} = (\xi_1 + w_{1d})(\xi_2 + w_{2d}) - \sqrt{6}(\xi_1 + w_{1d}) \\ \qquad -0.4(\xi_3 + w_{3d}) \end{cases} \tag{6}$$

The control problem is to find a control law u so that the state w can track any reference command $(\xi_1, \xi_2, \xi_3) \to 0$ when $t \to \infty$). The first Lyapunov function is defined as

$$V_1 = \frac{1}{2}\xi_1^{\,2} \tag{7}$$

First, a new variable, ξ, is introduced which moves the equilibrium point to the origin.

The derivative of the first Lyapunov function takes the Form:

$$\dot V_1 = \xi_1 \dot\xi_1 \tag{8}$$

$$\dot V_1 = -\xi_1\left(0.4(\xi_1 + w_{1d}) + \frac{\sqrt{6}}{6}(\xi_3 + w_{3d}) + \dot w_{1d}\right) \tag{9}$$

$$\dot V_1 < 0 \Rightarrow -0.4\,\xi_1^{\,2} - 0.4\,\xi_1 w_{1d} - \frac{\sqrt{6}}{6}\xi_1\xi_3$$

$$-\frac{\sqrt{6}}{6}\xi_1 w_{3d} - \xi_1 \dot w_{1d} < 0$$

To cancel the last four terms in the above derivative, we choose the update law:

$$-0.4\,\xi_1 w_{1d} - \frac{\sqrt{6}}{6}\,\xi_1\xi_3 - \frac{\sqrt{6}}{6}\,\xi_1 w_{3d} - \xi_1 \dot w_{1d} = 0$$

$$\xi_3 = -\frac{6}{\sqrt{6}}\left(0.4 w_{1d} + \frac{\sqrt{6}}{6}\, w_{3d} + \dot w_{1d}\right) \tag{10}$$

We take:

$$\alpha_1 = \xi_3 \tag{11}$$

with α_1 the Virtual control law.

Let $\acute\xi_3$ represent the error between ξ_3 and α_1:

$$\acute\xi_3 = \xi_3 - \alpha_1 \tag{12}$$

The derivative $\dot{\acute\xi}_3$ for the next design step is:

$$\dot{\acute{\xi}}_3 = \dot{\xi}_3 - \dot{\alpha}_1 \quad \text{With} \quad \dot{\xi}_3 = \dot{w}_3 - \dot{w}_{3d} \tag{13}$$

$$\dot{\acute{\xi}}_3 = \dot{w}_3 - \dot{w}_{3d} - \dot{\alpha}_1 \tag{14}$$

$$\dot{\acute{\xi}}_3 = (\xi_1 + w_{1d})(\xi_2 + w_{2d}) - \sqrt{6}(\xi_1 + w_{1d}) - 0.4(\acute{\xi}_3 + \alpha_1 + w_{3d}) - \dot{w}_{3d} - \dot{\alpha}_1 \tag{15}$$

Substituting the obtained virtual control rule (10) into (9), we obtain the first derivative of the Lyapunov function:

$$\dot{V}_1 = -0.4\,\xi_1^{\,2} \tag{16}$$

Step 2:

The second Lyapunov function takes the form (15) and (7)

$$V_2 = V_1 + \tfrac{1}{2}\acute{\xi}_3^{\,2} \quad \Rightarrow V_2 = \tfrac{1}{2}\xi_1^{\,2} + \tfrac{1}{2}\acute{\xi}_3^{\,2} \tag{17}$$

The time derivative of V_2 is:

$$\dot{V}_2 = \xi_1\dot{\xi}_1 + \acute{\xi}_3\dot{\acute{\xi}}_3 \tag{18}$$

$$\dot{V}_2 = -0.4\,\xi_1^{\,2} + \acute{\xi}_3\,[(\xi_1 + w_{1d})(\xi_2 + w_{2d}) - \sqrt{6}(\xi_1 + w_{1d}) - 0.4(\acute{\xi}_3 + \alpha_1 + w_{3d}) - \dot{w}_{3d} - \dot{\alpha}_1]$$

$$\dot{V}_2 = -0.4\,\xi_1^{\,2} - 0.4\acute{\xi}_3^{\,2} + \acute{\xi}_3[(\xi_1 + w_{1d})(\xi_2 + w_{2d}) - \sqrt{6}(\xi_1 + w_{1d}) - 0.4(\alpha_1 + w_{3d}) - \dot{w}_{3d} - \dot{\alpha}_1] \tag{19}$$

To cancel the last five terms in the above derivative, we choose the update law:

$$\xi_2 = \frac{\sqrt{6}(\xi_1 + w_{1d}) + 0.4(\alpha_1 + w_{3d}) + \dot{w}_{3d} + \dot{\alpha}_1}{\xi_1 + w_{1d}} - w_{2d} \tag{20}$$

We take:

$$\alpha_2 = \xi_2 \tag{21}$$

with α_2 the Virtual control law.

Let $\acute{\xi}_2$ represent the error between ξ_2 and α_2:

$$\acute{\xi}_2 = \xi_2 - \alpha_2 \tag{22}$$

The derivative $\acute{\xi}_2$ for the next design step is

$$\dot{\acute{\xi}}_2 = \dot{\xi}_2 - \dot{\alpha}_2 \tag{23}$$

$$\dot{\xi}_2 = \dot{w}_2 - \dot{w}_{2d} \tag{24}$$

$$\Rightarrow\ \dot{\acute{\xi}}_2 = \dot{w}_2 - \dot{w}_{2d} - \dot{\alpha}_2$$

$$\Rightarrow\ \dot{\acute{\xi}}_2 = -(\xi_1 + w_{1d})(\xi_3 + \alpha_1 + w_{3d}) + \frac{0.35}{2}\left(\acute{\xi}_2 + \alpha_2 + w_{2d}\right) + u - \dot{w}_{2d} - \dot{\alpha}_2 \tag{25}$$

Substituting the obtained virtual control rule (25) into (19), we obtain the second derivative of the Lyapunov function:

$$\dot{V}_2 = -0.4\,\xi_1^{\,2} - 0.4\acute{\xi}_3^{\,2} \tag{26}$$

Step 3:

The third Lyapunov function takes the form (17) and (20)

$$V_3 = V_2 + \tfrac{1}{2}\acute{\xi}_2^{\,2} \tag{27}$$

The time derivative of V_3 is :

$$\dot{V}_3 = \dot{V}_2 + \acute{\xi}_2\dot{\acute{\xi}}_2 \tag{28}$$

$$\dot{V}_3 = -0.4\,\xi_1^{\,2} - 0.4\acute{\xi}_3^{\,2} + \acute{\xi}_2[-(\xi_1 + w_{1d})(\xi_3 + w_{3d}) + \frac{0.35}{2}\left(\acute{\xi}_2 + \alpha_2 + w_{2d}\right) + u - \dot{w}_{2d} - \dot{\alpha}_2] \tag{29}$$

$$\dot{V}_3 = -0.4\,\xi_1^{\,2} - 0.4\acute{\xi}_3^{\,2} + \acute{\xi}_2[-(\xi_1 + w_{1d})(\xi_3 + w_{3d}) + \frac{0.35}{2}\left(\acute{\xi}_2 + \alpha_2 + w_{2d}\right) + u - \dot{w}_{2d} - \dot{\alpha}_2] \tag{30}$$

$$\dot{V}_3 = -0.4\,\xi_1^{\,2} - 0.4\acute{\xi}_3^{\,2} + \acute{\xi}_2\left[-(\xi_1 + w_{1d})(\xi_3 + w_{3d}) + \frac{0.35}{2}\left(\acute{\xi}_2 + \alpha_2 + w_{2d}\right) + u - \dot{w}_{2d} - \dot{\alpha}_2 - \beta\acute{\xi}_2 + \beta\acute{\xi}_2\right] \tag{31}$$

With β a positif constant.

$$\dot{V}_3 = -0.4\,\xi_1^{\,2} - 0.4\acute{\xi}_3^{\,2} + \acute{\xi}_2\left[-(\xi_1 + w_{1d})(\xi_3 + w_{3d}) + \frac{0.35}{2}\left(\acute{\xi}_2 + \alpha_2 + w_{2d}\right) + u - \dot{w}_{2d} - \dot{\alpha}_2 - \beta\acute{\xi}_2 + \beta\acute{\xi}_2\right] \tag{32}$$

$$\dot{V}_3 = -0.4\,\xi_1^{\,2} - 0.4\acute{\xi}_3^{\,2} - \beta\acute{\xi}_2 + \acute{\xi}_2\left[-(\xi_1 + w_{1d})(\xi_3 + w_{3d}) + \frac{0.35}{2}\left(\acute{\xi}_2 + \alpha_2 + w_{2d}\right) + u - \dot{w}_{2d} - \dot{\alpha}_2 + \beta\acute{\xi}_2\right] \tag{33}$$

so that $\dot{V}_3 < 0 \ \Rightarrow\ -(\xi_1 + w_{1d})(\xi_3 + w_{3d}) + \frac{0.35}{2}\left(\acute{\xi}_2 + \alpha_2 + w_{2d}\right) + u - \dot{w}_{2d} - \dot{\alpha}_2 + \beta\acute{\xi}_2 = 0$

From the third derivative given by (33), the control is determined as

$$u = (\xi_1 + w_{1d})(\xi_3 + w_{3d}) - \frac{0.35}{2}\left(\acute{\xi}_2 + \alpha_2 + w_{2d}\right) + \dot{w}_{2d} + \dot{\alpha}_2 - \beta\acute{\xi}_2 \tag{34}$$

Taking into account (3) and (23), (34) becomes:

$$u = w_1 w_3 - \frac{0.35}{2}w_2 - \beta w_2 + \dot{w}_{2d} + \dot{\alpha}_2 + \beta(w_{2d} + \alpha_2) \tag{35}$$

4. RESULTS AND DISCUSSION

In the numerical simulations, the fourth-order Runge–Kutta method is used to solve the systems with time step size 0.001 and we assume that the initial condition, $(w_1(0), w_2(0), w_3(0)) = (3\ 4.1\ 2)$.

We solve system (1) with the controller $u(t)$ as defined in (36). We put: $w_{1d} = w_{2d} = w_{3d} = 0 \Rightarrow \alpha_1 = \alpha_2 = 0$ and we choose $\beta = 1$.

.

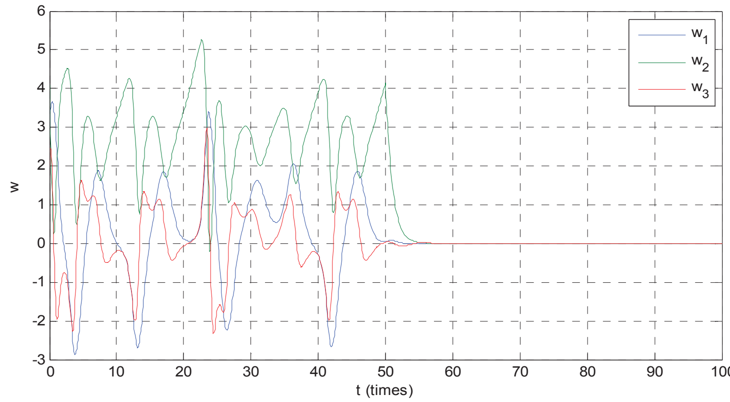

Fig. 2. Time series of the state variables (w_1, w_2, w_3) with $\beta = 1$.. The control input to origin is active at $t = 50$

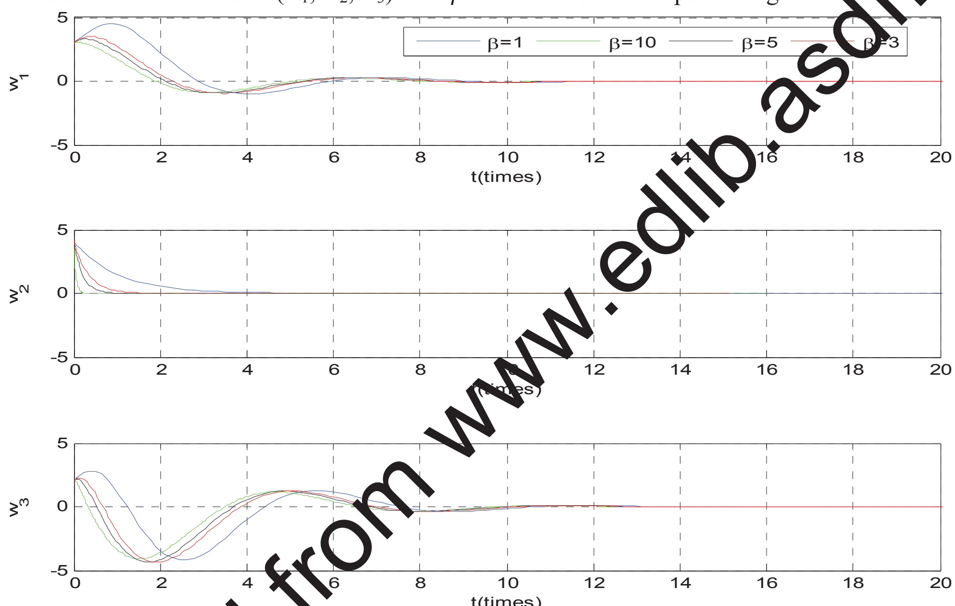

Fig. 3. Time series of the state variables (w_1, w_2, w_3) with different values of β.

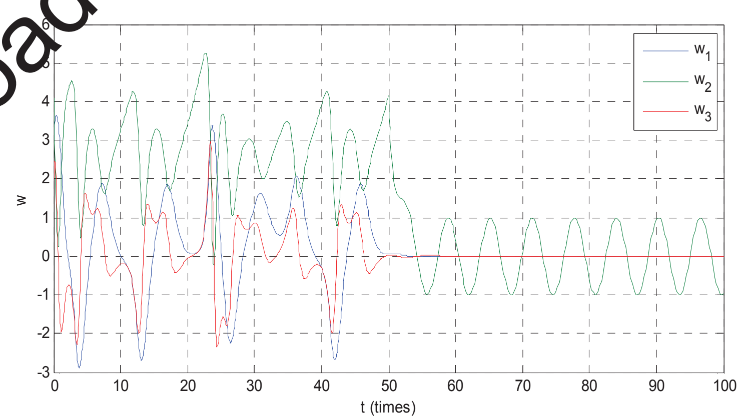

Fig. 4. Tracking response ($w_{1d}=w_{3d}=0$ $w_2=sin(t)$). The control input is active at $t = 50$.

ISBN : 978-81-925233-8-5

The results obtained in Figure 2. show that the state variables move chaotically with time when the controller is switched off and when the controller is active at t=50 the state variables are controlled to the origin (0, 0 ,0).

As seen form in Figure 3, whenever the value of beta increases the settling time of the system decrease. Figure 4. show the trajectory of the Chaotic Attitude Control of Satellite has been forced to the assigned orbit ($w_{1d}=w_{3d}=0$ and $w_2=sin(t)$).

5. CONCLUSION

In this paper, an effective control method for controlling Chaotic Attitude Control of Satellite has been proposed using active backstepping design. The numerical results obtained show that the backstepping control is effective in stabilizing the chaotic systems to a steady state as well as tracking of any desired trajectory to be achieved in a systematic way.

REFERENCES

[1] Chen, Mou; Wu, Qingxian; Jiang, Changsheng, Disturbance-observer-based robust synchronization control of uncertain chaotic systems, NONLINEAR DYNAMICS, vol. 70, no. 4, pp. 2421-2432, 2012

[2] L. Ling; Y. Miao, L. Ling, 'Synchronization between uncertain chaotic systems with a diverse structure based on a second-order sliding mode control', NONLINEAR DYNAMICS, vol. 70, no. 3, pp. 1861-1865, 2012

[3] Bessa, Wallace M.; de Paula, Aline S.; Savi, Marcelo A., " Sliding mode control with adaptive fuzzy dead-zone compensation for uncertain chaotic systems", NONLINEAR DYNAMICS, vol. 70, no. 3, pp. 1989-2001, 2012

[4] El-Dessoky, M.M. , Yassen, M.T, Adaptive feedback control for chaos control and synchronization for new chaotic dynamical system, Mathematical Problems in Engineering, vol. 2012, 2012

[5] V.N Belykh, N.F. Pedersen, H. Soerensen, Phys. Rev. B16 (1977) 4853.

[6] Sachin Bhalekar, "Chaos Control and Synchronization in Fractional-Order Lorenz-Like System," International Journal of Differential Equations, vol. 2012, Article ID 623234, 16 pages, 2012. doi:10.1155/2012/623234

[7] Harb, A.M et al. Bifurcation and chaos theory in electrical power systems: Analysis and control , Mathematical Problems in Engineering Volume 2012, 2012, Article number573910

[8] E. Ott, C. Grebogi, and J. A. Yorke, "Controlling chaos," Physical Review Letters, vol. 64, no. 11, pp. 1196–1199, 1990.

[9] K. Kemih, A. Kemiha, M. Ghanes. Chaotic attitude control of satellite using impulsive control. Chaos, Solitons & Fractals Volume 42, Issue 2, 30 October 2009, Pages 735–744.

[10] A. Mohammadbagheri, M. Yaghoobi, "Lorenz-Type Chaotic Attitude Control of Satellite through Predictive Control," cimsim, pp.147-152, 2011 Third International Conference on Computational Intelligence, Modelling & Simulation, 2011.

[11] U. E. Vincent, A. N. Njah, and J.A. Laoye: Controlling chaos and deterministic directed transport in inertia ratchets using backstepping control, Physical D 231 (2007) 130.

[12] Vincent, U. E., Ucar, A., Laoye, J. A., and Kareem, S. O. (2008), Control and Synchronization of chaos in RCL-shunted Josephson junction using backstepping design, Physica C, 468: 374-382.

[13] Yassen, M.T, Controlling, synchronization and tracking chaotic Liu system using active backstepping design, Physics Letters, Vol. 360, Issue 4-5, 8 January 2007, Pages 582-587

[14] A.P.M. Tsui, A.J. Jones, The control of higher dimensional chaos: comparative results for the chaotic satellite attitude control problem, Physica D, V.135, 2000, 41-62.