Stabilizing Periodic Orbits of Chaotic System Using Adaptive Type-2 Fuzzy Sliding Mode Control

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***Abstract*—In this paper, a novel adaptive type-2 fuzzy sliding mode control is designed. In order to stabilize the unstable periodic orbits of uncertain perturbed chaotic system. This letter is assumed to have an affine form with unknown mathematical model, the type-2 fuzzy system is used to overcome this constraint. For sliding mode, adaptive fuzzy type 2 systems have been introduced in order to generate the switching signal to avoid both the chattering and the constraint on the knowledge of upper bounds disturbances and uncertainties. These adaptive fuzzy type-2 systems are adjusted on-line by adaptation laws deduced from the stability analysis in Lyapunov sense. Simulation results show the good tracking performances by using the proposed approach.**

Keywords-- Chaotic system, sliding mode, fuzzy logic type-2; Lyapunov stability.

I. INTRODUCTION

Chaos is a particular case of nonlinear dynamics that has some specific characteristics such as extraordinary sensitivity to initial conditions and system parameter variations, it is well known fact that chaotic dynamics exist in a large variety of nature systems (e.g, aerodynamics, biological and physical systems). Many nonlinear control techniques have been applied for chaos elimination and chaos synchronization such as active control, adaptive control, sliding mode control and fuzzy control [1-8].

A useful and effective control scheme to deal with uncertainties, time varying properties, nonlinearities and bounded externals disturbances is the sliding mode control (SMC) [9-11]. However, its major drawback in practical applications is the chattering problem.

Numerous techniques have been proposed to eliminate this phenomenon in SMC as boundary layer method, fuzzy logic approach and higher order sliding mode [12-16].

The objective of this paper is to force the *n*-dimensional chaotic system to a desired state even if it has uncertainties system and external disturbances, by incorporation the fuzzy type-2 approach and sliding mode control. We introduced adaptive type-2 fuzzy systems for model the unknown dynamic of system and calculate the switching term of sliding mode controller. Their updates are performed using adaptation laws derived from the study of stability in the Lyapunov sense.

The organization of this paper is as follows. After a problem formulation In section II, we give brief description of Interval type-2 fuzzy logic system in section III. In section IV, the adaptive type-2 fuzzy sliding mode control scheme is presented. Simulation example demonstrate the efficiently of the proposed approach in section V. Finally, section VI gives the conclusions of the advocated design methodology.

II. PROBLEM FORMULATION

Consider a chaotic n^{th} order uncertain system which has an affine form:

$$\begin{cases} \dot{x}_i = x_{i+1}, \quad 1 \le i \le n-1, \\ \dot{x}_n = f(\underline{x},t) + d(t) + u(t), \end{cases} \quad \underline{x} = [x_1(t)\ x_2(t) \ldots x_n(t)] \in \Re^n \qquad (1)$$

where $\underline{x}$ is the measurable state vector, $f(\underline{x},t)$ is unknown nonlinear continuous and bounded function, and $u(t) \in \Re$ state output and control input of the system, respectively. $d(t)$ is the external bounded disturbance,

$$\begin{cases} |f(\underline{x},t)| < F \\ |d(t)| \le \Delta_d \end{cases} \qquad (2)$$

where F, Δ_d are two positive constants.

The control problem is to get the system to track an *n*-dimensional desired vector $\underline{y}_d(t)$ which belong to a class of continuous functions on $[t_0, \infty]$. Let's the tracking error as;

$$\begin{aligned} \underline{e}(t) &= \underline{x}(t) - \underline{y}_d(t) \\ &= [x(t) - y_d(t)\ \dot{x}(t) - \dot{y}_d(t) \ldots x^{(n-1)}(t) - y_d^{(n-1)}(t)] \\ &= [e(t)\ \dot{e}(t) \ldots e^{(n-1)}(t)] = [e_1(t)\ e_2(t) \ldots e_n(t)], \end{aligned} \qquad (3)$$

The control goal considered is that;

$$\lim_{t\to\infty} \|\underline{e}(t)\| = \lim_{t\to\infty} \|\underline{x}(t) - \underline{y}_d(t)\| \to 0, \qquad (4)$$

To satisfy this objective, we consider the following sliding surface [17]:

$$s(\underline{e},t) = \left(\frac{\partial}{\partial t} + \lambda\right)^{(n-1)} \underline{e} \qquad (5)$$

where λ is a positive constant. It defines the slop of sliding surface. The derivative of s is:

$$\dot{s}(\underline{e},t) = e^{(n)} + \delta_s \qquad (6)$$

where $\delta_s = \sum_{k=1}^{n} \frac{(n-1)!}{k!(n-k-1)!} \left(\frac{\partial}{\partial t}\right)^{(n-k-1)} \lambda^k \underline{\dot{e}}$. By using (1) we obtain:

$$\dot{s}(\underline{e},t) = \delta_s + y_d^{(n)} - f(\underline{x},t) - u(t) - d \qquad (7)$$

If $f(\underline{x},t)$ is known and free of external disturbances and uncertainties, and when the system is restricted to the sliding surface $s(\underline{x},t) = 0$, it will be governed by an equivalent control u_{eq} obtained by:

$$u_{eq} = \left[-f(\underline{x},t) + \delta_s + y_d^{(n)}\right] \qquad (8)$$

The global control is composed of the equivalent control and the switching term u_s such that:

$$u_s = -k\, sign(s(\underline{e},t))$$

By adding this term to (8), we obtain the global control:

$$u=\left[-f\,(\underline{x},t)+\delta_s+y_d^{(n)}+u_s\right] \tag{9}$$

The sufficient condition to ensure the transition trajectory of the tracking error from approaching phase to the sliding one is:

$$\frac{1}{2}\frac{d}{dt}s^2(\underline{e},t)=s(\underline{e},t)\dot{s}(\underline{e},t)\le-\eta\left|s(\underline{e},t)\right| \tag{10}$$

After some manipulations, we obtain:

$$-k+d\,sign\left(s(\underline{e},t)\right)\le-\eta \tag{11}$$

Then we can choose the parameters of k as follows:

$$k\ge\eta+|d| \tag{12}$$

Note that the control law (9) depends only on the parameters λ, k, and nonlinear continuous function $f\,(\underline{x},t)$. However, in the approaching phase, the knowledge of the $d's$ upper bound and $f\,(\underline{x},t)$ is required in the optimal choice of k. Therefore $f\,(\underline{x},t)$ is unknown and $d(t)\ne 0$.

In the rest of paper we solved these problems by introducing an adaptive fuzzy sliding mode controller.

The purpose of this paper is to approximate $f\,(\underline{x},t)$ and the switching control by interval type-2 fuzzy systems. Furthermore, the adaptive laws will be derived to adjust parameters.

III. INTERVAL TYPE-2 FUZZY LOGIC SYSTEM

Fuzzy Logic Systems (FLSs) are known as the universal approximators and have various applications in identification and control design. A type-1 fuzzy system consists of four major parts: fuzzifier, rule base, inference engine, and defuzzifier. A T2FLS is very similar to a T1FLS [18]; the major structure difference being that the defuzzifier block of a T1FLS is replaced by the output processing block in a T2FLS, which consists of type-reduction followed by defuzzification.

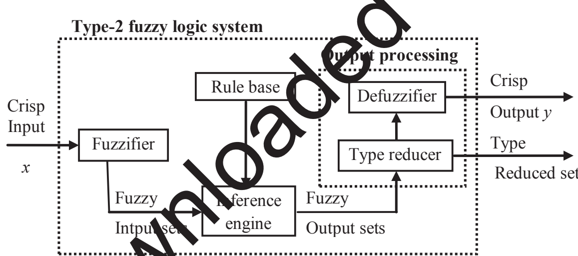

Figure 1. Structure of a type-2 fuzzy logic system.

In a T2FS, a Gaussian function with a known standard deviation is chosen, while the mean (m) varies between $m1$ and $m2$. Therefore, a uniform weighting is assumed to represent a footprint of uncertainty as shaded in Figure. 2. Because of using such a uniform weighting, we name the T2FS as an Interval Type-2 Fuzzy Set (IT2FS).

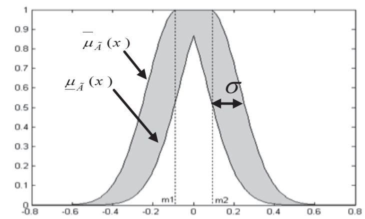

Figure 2. Interval type-2 Gaussian fuzzy set.

It is obvious that the type-2 fuzzy set is in a region bounded by an upper MF and a lower MF denoted as $\bar{\mu}_{\tilde{A}}(x)$ and $\underline{\mu}_{\tilde{A}}(x)$ respectively, and is called a foot of uncertainty (FOU). Assume that there are M rules in a type-2 fuzzy rule base, each of which has the following form:

$$R^i:\ IF\ x_1\ is\ \tilde{F}_1^i, and\ \ldots, and\ x_n\ is\ \tilde{F}_n^i\ , THEN\ y\ is\left[w_l^i\ w_r^i\right]$$

where x_j, $j=1,2,\ldots,n$ and y are the input and output variables of the type-2 fuzzy system, respectively, the $\tilde{F}_n^i$ is the type-2 fuzzy sets of antecedent part, and $\left[w_l^i\ w_r^i\right]$ is the weighting interval set in the consequent part. The inference engine combines rules and provides a mapping from input type-2 fuzzy sets to output type-2 fuzzy sets. The operation of type-reduction is to give a type-1 set from a type-2 set.

In the meantime, the firing strength F^i for the ith rule can be an interval type-2 set expressed as;

$$F^i=[\underline{f}^i, \bar{f}^i] \tag{13}$$

where

$$\begin{aligned}\underline{f}^i&=\underline{\mu}_{\tilde{F}_1^i}(x_1)*\ldots*\underline{\mu}_{\tilde{F}_n^i}(x_n)\\ \bar{f}^i&=\bar{\mu}_{\tilde{F}_1^i}(x_1)*\ldots*\bar{\mu}_{\tilde{F}_n^i}(x_n)\end{aligned} \tag{14}$$

In this paper, the center of set type-reduction method [18] is used to simplify the notation. Therefore, the output can be expressed as;

$$\begin{aligned}y_{\cos}(x)&=\left[y_l, y_r\right]\\ &=\int_{w^1\in\left[w_l^1\ w_r^1\right]}\cdots\int_{w^M\in\left[w_l^M\ w_r^M\right]}\\ &\quad\times\int_{f^1\in\left[\underline{f}^1\ \bar{f}^1\right]}\cdots\int_{f^M\in\left[\underline{f}^M\ \bar{f}^M\right]} 1\Bigg/\frac{\sum_{i=1}^{M}f^iw^i}{\sum_{i=1}^{M}f^i}\end{aligned} \tag{15}$$

where $y_{\cos}(x)$ is also an interval type-1 set determined by left and right most points (y_l and y_r), which can be derived from consequent centriod set $[w_r^i, w_l^i]$ (either $\underline{w}^i$ or $\bar{w}^i$) and the firing strength $f^i\in F^i=\left[\underline{f}^i, \bar{f}^i\right]$. The interval set $[w_r^i, w_l^i]$ ($i=1,\ldots,M$) should be computed or set first before the computation of $y_{\cos}(x)$. For any value $y\in y_{\cos}$. Hence, left-most point y_l and right-most point y_r can be expressed as [18];

$$y_l=\frac{\sum_{i=1}^{M}f_l^iy_l^i}{\sum_{i=1}^{M}f_l^i}\quad\text{and}\quad y_r=\frac{\sum_{i=1}^{M}f_r^iy_r^i}{\sum_{i=1}^{M}f_r^i} \tag{16}$$

The defuzzified crisp output from an IT2FLS is the average of y_r and y_l that is:

$$y(x) = \frac{y_l + y_r}{2} \tag{17}$$

IV. ADAPTIVE INTERVAL TYPE-2 FUZZY SECOND ORDER SLIDING MODE CONTROL

In this section, the nonlinear function $f(\underline{x},t)$ and switching signal u_s will be replaced by adaptive type-2 fuzzy systems. Furthermore, the adaptive laws to adjust parameters will be derived from the stability study of the closed-loop process in the Lyapunov sense. This adaptation in addition smoothing the control will enable us to better anticipate disturbances.

A. Proposed Type-2 fuzzy logic systems (FLS)

All adaptive systems used to approximate the control gains and unknown function $f(\underline{x},t)$ have the same structure presented in (15), as the output fuzzy systems using the center of set method [19]:

$$Y_{\cos}(\theta^1,\ldots\theta^M, f^1,\ldots,f^M) = [y_l, y_r]$$

$$= \int_{\theta^1}\cdots\int_{\theta^M}\int_{f^1}\cdots\int_{f^M} 1 \Big/ \frac{\sum_{i=1}^{M} f^i\theta^i}{\sum_{i=1}^{M} f^i} \tag{18}$$

Hence, left-most point y_l and right-most point y_r can be expressed as;

$$y_l = \frac{\sum_{i=1}^{M} f_l^{\,i}\theta_l^i}{\sum_{i=1}^{M} f_l^{\,i}} \quad \text{and} \quad y_r = \frac{\sum_{i=1}^{M} f_r^{\,i}\theta_r^i}{\sum_{i=1}^{M} f_r^{\,i}} \tag{19}$$

where θ^i , i=1,...M are the adjustable parameters. The defuzzified crisp output from an IT2FLS is the average of y_r and y_l as in (16).

We replace $f(\underline{x},t)$ and the switching term by $\hat{f}(\underline{x},\underline{\theta}_f)$ and $\hat{u}_s(s,\underline{\theta})$ respectively such that:

$$\hat{f}(\underline{x},\underline{\theta}_f) = \underline{\theta}_f^{\,T}\,\underline{\xi}_f(\underline{x}) \tag{20}$$

$$\hat{u}_s(s,\underline{\theta}) = \underline{\theta}^T\,\underline{\xi}(s) \tag{21}$$

where $\underline{\theta}_f$ *and* $\underline{\theta}$ are adjustable parameters vectors.

B. Proposed controller synthesis

In order to guarantee the global stability of closed loop system (1) with the convergence of tracking error to zero, we propose the following control law:

$$u = \left[-\hat{f}(\underline{x},t) + \delta_s + y_d^{(n)} + \hat{u}_s(s)\right] \tag{22}$$

In order to derive the adaptive laws of adjusting $\underline{\theta}_f$ *and* $\underline{\theta}$. We define the optimal parameter vector $\underline{\theta}_f^*$ and $\underline{\theta}^*$ as;

$$\underline{\theta}_f^* = \arg\min_{\underline{\theta}_f\in\Omega_f}\left[\sup_{x\in\Omega_x}\left|\hat{f}(\underline{x},\underline{\theta}_f) - f(\underline{x},t)\right|\right],$$

$$\underline{\theta}^* = \arg\min_{\underline{\theta}\in\Omega}\left[\sup_{s\in\Omega_s}\left|\hat{u}(s,\underline{\theta}) - u_s\right|\right].$$

where $\Omega_f, \Omega, \Omega_x$ and Ω_s are constraint sets of suitable bounds on θ_f, θ, x and *s*, respectively and they are defined as;

$$\Omega_f = \left\{\underline{\theta}_f : \left|\underline{\theta}_f\right| \le M_f\right\},\ \Omega = \left\{\underline{\theta} : \left|\underline{\theta}\right| \le M\right\},$$

$$\Omega_x = \left\{x : |x| \le M_x\right\},\quad \Omega_s = \left\{s : |s| \le M_s\right\},$$

where M_f, M , M_x and M_s are positive constants.

The minimum approximation error is defined as;

$$w = \left[f(\underline{x},t) - \hat{f}(\underline{x},\underline{\theta}_f^*)\right]$$

We can write,

$$|w| \le \left|f(\underline{x},t) - \hat{f}(\underline{x},\underline{\theta}_f^*)\right|$$

$$\le \left|f(\underline{x},t)\right| + \left\|\underline{\theta}_f^{*T}\right\|\left\|\underline{\xi}_f(\underline{x})\right\|$$

$$\le F + M_f$$

By using $F + M_f = \beta$, it can be easily concluded that w is bounded $w \le \beta$.

Then the optimal parameters of $f(\underline{x},t)$ and u_s are defined as:

$$\hat{f}(\underline{x},\underline{\theta}_f^*) = \underline{\theta}_f^{*T}\,\underline{\xi}_f(\underline{x}) \tag{23}$$

$$\hat{u}_s^*(s,\underline{\theta}^*) = \underline{\theta}^{*T}\,\underline{\xi}(s) \tag{24}$$

To study the closed loop stability and to find the adaptation laws of adjustable parameters, we consider the following Lyapunov function:

$$V(s,\tilde{\underline{\theta}}) = \frac{1}{2}s^2 + \frac{1}{2\gamma_f}\tilde{\underline{\theta}}_f^{\,T}\tilde{\underline{\theta}}_f + \frac{1}{2\gamma}\tilde{\underline{\theta}}^T\tilde{\underline{\theta}} \tag{25}$$

where $\tilde{\underline{\theta}} = \underline{\theta} - \underline{\theta}^*$ and $\tilde{\underline{\theta}}_f = \underline{\theta}_f - \underline{\theta}_f^*$. γ_f and γ are positive training constants, so the time derivative of (25) is :

$$\dot{V}(s,\tilde{\underline{\theta}}) = \dot{s}(\underline{e},t)s(\underline{e},t) + \frac{1}{\gamma_f}\tilde{\underline{\theta}}_f^{\,T}\dot{\underline{\theta}}_f + \frac{1}{\gamma}\tilde{\underline{\theta}}^T\dot{\underline{\theta}} \tag{26}$$

By using the control law (22), the equation (20-21), the time derivative of the sliding surface (7) becomes:

$$\begin{aligned}\dot{s}(\underline{e},t) &= f(\underline{x},t) + d - \hat{f}(\underline{x},\underline{\theta}_f^*) + \hat{u}_s(s)\\ &= f(\underline{x},t) - \hat{f}(\underline{x},\underline{\theta}_f) + \hat{f}(\underline{x},\underline{\theta}_f^*) - \hat{f}(\underline{x},\underline{\theta}_f^*)\\ &\quad + \hat{u}_s(s) + \hat{u}_s^*(s) - \hat{u}_s^*(s) + d\\ &= w - (\underline{\theta}_f - \underline{\theta}_f^*)^T\,\underline{\xi}_f(\underline{x}) + (\underline{\theta} - \underline{\theta}^*)^T\,\underline{\xi}(s) + \hat{u}_s^*(s) + d\end{aligned} \tag{27}$$

The substitution of (27) in (26) gives:

$$\begin{aligned}\dot{V} &= s(\underline{e},t)\left(w + \hat{u}_s^*(s) + d\right)\\ &\quad + \frac{1}{\gamma_f}\tilde{\underline{\theta}}_f\left(\dot{\underline{\theta}}_f - \gamma_f s(\underline{e},t)\underline{\xi}_f(\underline{x})\right)\\ &\quad + \frac{1}{\gamma}\tilde{\underline{\theta}}\left(\dot{\underline{\theta}} + \gamma\, s(\underline{e},t)\underline{\xi}(s)\right)\end{aligned} \tag{28}$$

By choosing the following adaptation laws:

$$\dot{\underline{\theta}}_f = \gamma_f s(\underline{e},t)\underline{\xi}_f(\underline{x}) \tag{29}$$

$$\dot{\underline{\theta}} = -\gamma\, s(\underline{e},t)\underline{\xi}(s) \tag{30}$$

where $\dot{\tilde{\underline{\theta}}} = \dot{\underline{\theta}}$ and $\dot{\tilde{\underline{\theta}}}_f = \dot{\underline{\theta}}_f$. Therefore, we obtain:

$$\dot{V} = s\left(w + d - k^*\,sign(s)\right) \tag{31}$$

$$\begin{aligned}\dot{V} &= ws + ds - k|s| \\ &\leq -\eta|s| + |w||s| \\ &\leq -\eta + \beta\end{aligned} \tag{32}$$

According to Barbalat's lemma [17], we can state that the sliding surfaces are constructed to be attractive and $\lim_{t\to\infty} \underline{e}(t) = 0$. Therefore, the control objective is satisfied, and hence, we can synthesize the robust controller based on sliding mode type-2 fuzzy controller, in which we can force the output system $\underline{x}$ to follow a bounded reference trajectory $\underline{y}_d$.

The overall scheme of the adaptive type-2 fuzzy sliding mode control is shown in Figure. 3.

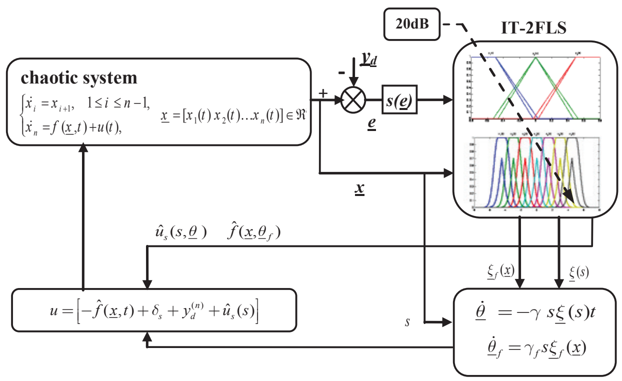

Figure 3. Overall adaptive type-2 fuzzy sliding mode control scheme in presence of noise.

V. SIMULATION EXAMPLE

The above described control scheme is now used to control the states of modified duffing system. Consider the following duffing system, which may exhibit chaotic behavior. The dynamic equation of system is given by;

$$\begin{cases}\dot{x}_1 = x_2, \\ \dot{x}_2 = -p_1 x_2 + p_2 x_1 - p_3 x_1^3 + q\cos(wt),\end{cases} \tag{33}$$

where $p_1 = 0.4$, $p_2 = 1.1$, $p_3 = 1$, $q = -2.1$ and $w = 1.8$. The simulation results of the system (33) free of input are shown in (Figure.4) with initial states $x_1(0)=1$ and $x_2(0)=0$.

The control objective is to force the states $x_i(t), i = 1, 2$ of the system (33) to track the reference trajectories $y_{d1}(t) = (\pi/3)(\sin(t) + 0.3\sin(3t))$, and $y_{d2}(t) = (\pi/3)(\cos(t) + 0.9\cos(3t))$. The system initial conditions are $x(0) = [1 \quad 0]^T$ and the proposed control input signal is (22), then the system (33) can be expressed by;

$$\begin{cases}\dot{x}_1 = x_2, \\ \dot{x}_2 = -p_1 x_2 + p_2 x_1 - p_3 x_1^3 + q\cos(wt) + u(t),\end{cases}$$

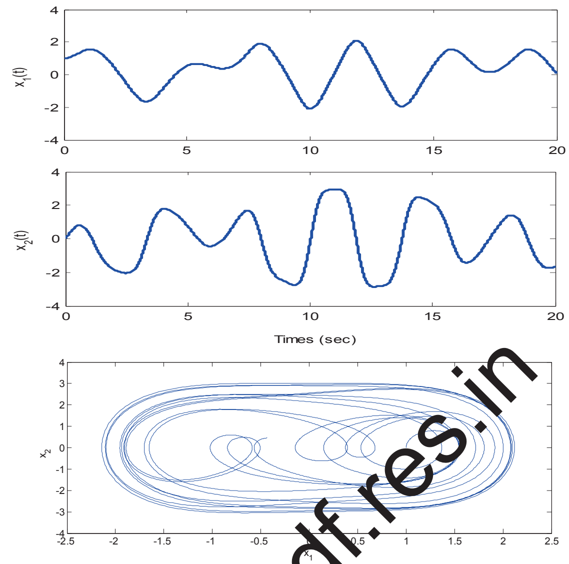

Figure 4. Time response (x1, x2) and typical chaotic behavior of duffing oscillator.

To design the equivalent part of control signal, the input variables of the fuzzy system $\hat{f}(\underline{x}, \underline{\theta}_f)$ are chosen as $x_i(t), i = 1, 2$. The following type-1 and interval type-2 fuzzy membership functions of $x_i(t), i = 1, 2$ are selected as $\mu_{F_i^l}(\underline{x})$ for $l=1,\ldots,7$, shown in TABLE.I, with variance $\sigma = 0.5$ and initial values $\theta_f(0) = O_{2*7}$. Similarly to generate the adaptive fuzzy system which allow us to approximate the reaching part of control signal (u_s), we consider three type-2 fuzzy interval sets according to the variable $s(t)$ (Figure. 5).

The sliding surface is selected as: $s = \dot{e} + \lambda e$; where $\lambda = 10$. Simulation time *tf* = 20 second and the step size *h=0.005*.

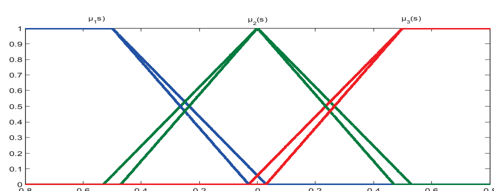

Figure 5. Interval type-2 antecedent membership functions of $s(t)$.

TABLE I. INTERVAL TYPE-2 AND TYPE -1 FUZZY MEMBERSHIP FUNCTIONS FOR x_i *(i= 1, 2)*.

	Mean (*m*)				Mean (*m*)		
	m_1	m_2	*m* (type-1)		m_1	m_2	*m* (type-1)
$\mu_{F_i^1}(x_i)$	-3.5	-2.5	-3	$\mu_{F_i^5}(x_i)$	0.5	1.5	1
$\mu_{F_i^2}(x_i)$	-2.5	-1.5	-2	$\mu_{F_i^6}(x_i)$	1.5	2.5	2
$\mu_{F_i^3}(x_i)$	-1.5	-0.5	-1	$\mu_{F_i^7}(x_i)$	2.5	3.5	3
$\mu_{F_i^4}(x_i)$	-0.5	0.5	0				

The simulation results are classified into two parts as follows: adaptive type-1 fuzzy controller and adaptive type-2 fuzzy controller. For the both simulations we choose the initial system states $x(0) = [1 \quad 0]^T$, and the training data is corrupted with internal noise (SNR=20dB).

Part I: Adaptive type-1 Fuzzy sliding mode control;

We see in figures. 6, 7, that the state trajectories *($x_1(t)$, $x_2(t)$)* converge quickly to their references *($y_{d1}(t)$, $y_{d2}(t)$)*, Figure. 8 shows the applied control.

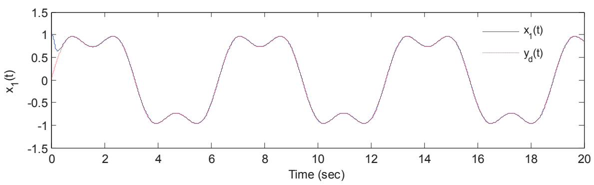

Figure 6. The output trajectory of $x_1(t)$.

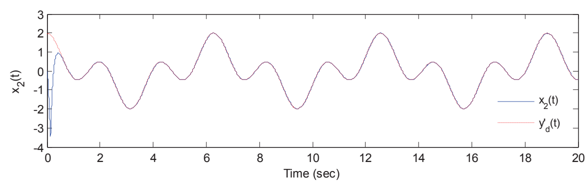

Figure 7. The output trajectory of $x_2(t)$.

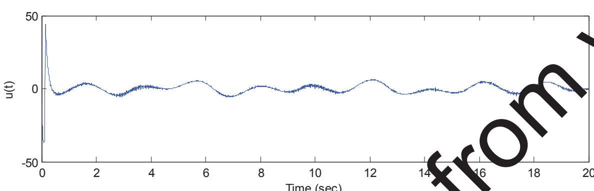

Figure 8. Applied control $u(t)$.

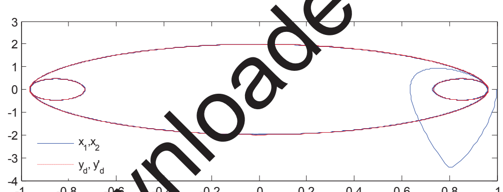

Figure 9. Phase-plane trajectory of system.

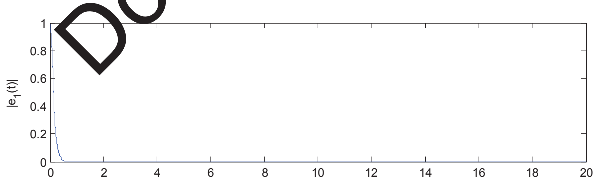

Figure 10. The absolute tracking error $|e_1(t)|$.

Part II: Adaptive type-2 Fuzzy sliding mode control;

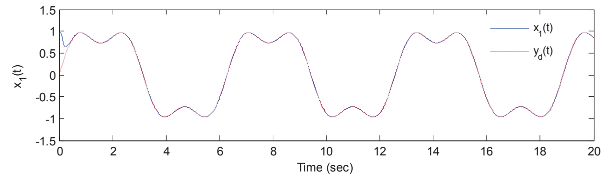

Figure 11. The output trajectory of $x_1(t)$.

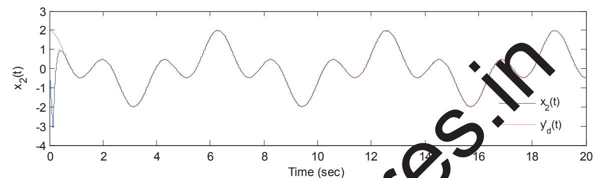

Figure 12. The output trajectory of $x_2(t)$.

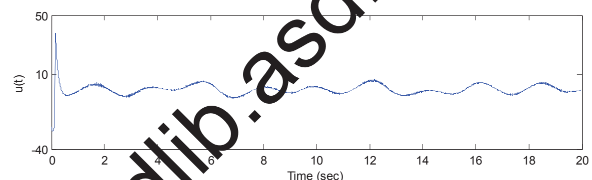

Figure 13. Applied control $u(t)$.

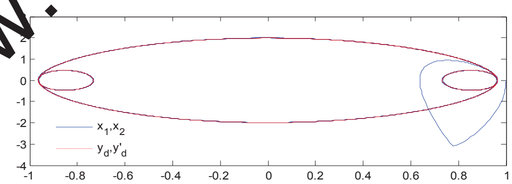

Figure 14. Phase-plane trajectory of system

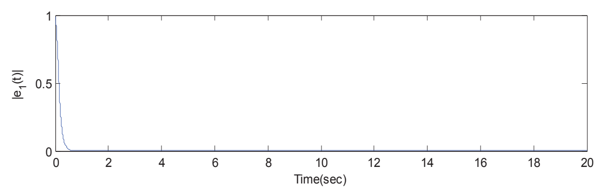

Figure 15. The absolute tracking error $|e_1(t)|$.

According to figures 11, 12 and 15, we see a good tracking reference of *($x_1(t)$, $x_2(t)$)*. Figure. 13 shows the applied control.

In order to have a quantitative comparison of tracking error and control effort, in (TABLE.II), we have reported the Integral Absolute Error (IAE) of the both controller.

We clearly show that the proposed approach (adaptive type-2 fuzzy sliding mode controller) provides a better tracking performance than that of type-1 fuzzy controller and this latter must expend more control effort. Nevertheless, from above simulation results, we can see that in order to deal with noisy training data the type-1 fuzzy controller must expend more control effort.

TABLE II. COMPARISON OF CONTROL EFFORTS AND INTEGRAL ABSOLUTE ERROR (IAE) VALUES OF TOW CONTROLLERS.

T = tf / h = 4000			
		Control effort $\sum_{i=1}^{T} \lvert u_i \rvert$	**IAE** $\int_0^{20} \lvert e_1(t) \rvert dt$
20 dB Noise	Type-1	10444	2.2372
	Type-2	10397	2.2187

VI. CONCLUSION

In this paper, the problem of stabilization orbit of nonlinear uncertain chaotic system in the presence of internal disturbance is solved by incorporation of adaptive interval type-2 control scheme and sliding mode approach. In order to eliminate the chattering phenomenon efficiently, adaptive interval type-2 fuzzy systems are introduced to calculate the switching control term. Based on the Laypunov stability criterion, the adaptation laws of adjustable parameters of the type-2 fuzzy systems and the stability of closed loop system are ensured. A simulation example has been presented to illustrate the effectiveness and the robustness of the proposed approach.

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