

Single Tree Search Sphere Decoding Algorithm for Mimo Communication System

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Abstract— Multiple Input Multiple Output (MIMO) systems use multiple antennas at both transmitter and receiver for higher Bandwidth efficiency. The implementation of MIMO detection becomes a difficult task as the computational complexity increases with the number of transmitting antenna and constellation size increases. The decoder for a 4*4 MIMO system with 16-QAM modulation and spatial multiplexing is implemented using Mat lab. Difficult part is MIMO detection; ML decoding cannot be implemented directly because increase the complexity exponentially if size of constellation and number of transmit antenna increases. Sphere decoding reduce the complexity of decoding with some improvement with the decoding rate and BER. In my project sphere decoding combined with single tree search and ML decoding greatly improve the decoding Rate and BER. In sphere decoding selecting the sphere radius is very important. Sphere decoding algorithm implemented in the tree search complexity of algorithm is more reduced. In tree search Tree pruning strategies are used to reduce the complexity of the tree search based algorithms. The basic idea is to reduce the number of tree nodes visited to achieve a ML result. The decision where to visit a node or prune it is based on its Partial Euclidean Distance (PED). Depending upon tree pruning strategy the algorithm achieve optimal BER.

Index Terms— Lattice point search, Multiple Input Multiple Output (MIMO) detection, Quadrature Amplitude Modulation, Sphere decoding, partial Euclidean distance (PED), Log Likelihood Ratio (LLR).

I. INTRODUCTION

With continuous need of higher communication rate within a fixed frequency spectrum, Multiple Input Multiple Output technology provide a potential solution for saving the frequency spectrum. Every new wireless communication protocol like WiMAX, Wi-Fi, LTE etc. is employing MIMO technology to satisfy ever increasing demand of high data rate. In MIMO systems multiple antennas are employed on both transmitter and receiver side. Spatial multiplexing method is applied to increase the data rate. In spatial multiplexing higher rate input data stream is sub-divided into lower rate sub data streams. These data streams are then propagated from multiple antennas. It has a potential of increasing the capacity N times, where N is the minimum of the total number of transmit and receive antennas. On the receiver side received signal at each antenna corresponds to a combination of multiple data streams from all the transmit antennas. In spatial multiplexing the input data stream with high data rate is split into multiple smaller data rate bit streams. These separate data streams are then further modulated in which a set of bits are assigned a fixed codeword in the symbol constellation. The advantages of MIMO systems are more. A typical MIMO system can be used to increase link reliability and Quality of Service (QoS) by using spatial Diversity Gain methods. Using this method the input stream is space time coded and then transmitted which gives more robustness but data rate is lower. MIMO system can also be used in multiplexing gain mode where each transmit antenna sends out different bit streams hence throughput of the MIMO system is increased. Fig 1 shows system model of MIMO communication system.

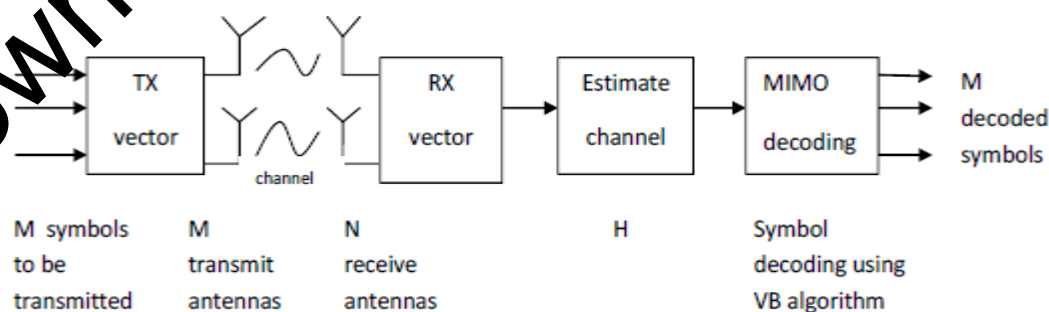


Fig 1. System Model

II. RELATED WORK

Decoding part is heart of the MIMO communication system. In MIMO decoding several detectors are available they are (i) Zero forcing (ZF) (ii) MMSE (Minimum Mean square error) (iii) Successive interference cancellation (SIC) detector (iv) Sphere decoding with Maximum Likelihood (ML). Zero forcing and MMSE Detector coming under the family of reduced complexity detector is linear detector. Even though they reduce the computational complexity, their drawback is significant performance degradation. SIC detector are prone to error propagation. All the drawback is overcome by sphere decoding using ML. Sphere decoding have the ability to reach ML solution at lower complexity than exhaustive search, by finding the result within a sphere centered at receive signal.

A. Sphere decoding algorithm

Considering a MIMO system with M transmit antenna and N receive antennas, the received signal is given by the equation

$$X = Hs + v \quad (1)$$

Where 'X' is a received signal vector and 'v' is an additive white Gaussian noise vector (AWGN). 'H' is the N*M channel matrix that can be assumed as known from perfect channel estimation. In each element of the transmit vector, 's' is a constellation point and the channel matrix 'H' generates a lattice for the selected modulation scheme. Where size of the signal are $X = [x_1, x_2, x_3, \dots, x_{N_r}]$, $S = [s_1, s_2, s_3, \dots, s_{M_t}]$ and $v = [v_1, v_2, v_3, \dots, v_{N_r}]$.

1. ML decoding algorithm

In ML decoding algorithm is to find the minimum distance between the received point (X) and the examining lattice point (Hs) that

$$\text{Estimated } s = \arg \min \|X - Hs\|_2 \quad (2)$$

$$S \in \Omega_{M_t}$$

Where 's' is the decoded vector. The entries of 's' are chosen from a complex constellation 'Ω'. The set of all possible transmitted vector symbols is denoted by 'Ω_{Mt}'.

Here one drawback is exhaustive search. If M_t=2 and Q=16 (Number of constellation points) then search space = Ω_{Mt} = 16² = 256 MIMO symbol, that is to find one symbol using ML decoding has to search 256 symbol and find the one symbol out of 256 MIMO symbols having less distance that symbol is correspond to required symbol. If M_t=4 and Q=16 (Number of constellation points) then search space = Ω_{Mt} = 16⁴ = 65,536, that is to find one symbol out of 65536 MIMO symbol, ML decoding has to compare 65,536 symbol and find the one symbol having less distance that is expected correct symbol. So practically not possible, the searching time is more definitely the decoding rate is reduced. Thus, the ML-based sphere decoding system can be summarized as follows,

Input: The input is the channel lattice generation matrix H and the received signal vector X.

Output: A 1*M vector such that Hs is a lattice point that is the closest to 'X'.

2. Sphere decoding

In sphere decoding

$$\text{Estimated } s = \arg \min \|X - Hs\|_2 < D_2 \quad (2)$$

$$s \in \Omega_{M_t}$$

For simplicity we assume that M=N then Maximul likelihood solution in (2) can be written as

$$N_t N_t$$

$$\text{Estimated } s = \arg \min \left\{ \sum_{k=1}^{N_t} |x_k - \sum_{i=1}^{N_t} h_{ik} x_k|^2 \right\} \quad (3)$$

$$S \in \Omega_{M_t} \quad i=1 \quad k=i$$

From the received signal X as centre point and take some radius d make a circle that form a sphere with in this sphere find distance between the received point (X) and constellation points inside the sphere already created in the receiver using Hs, from the different distances which gives the minimum distance then corresponding constellation point created at the receiver is gives correct recovered symbol. Choosing Radius is very difficult, if sphere radius is very small no QAM symbol available for compare, If sphere diameter is large then more symbol available for comparison then It is like ML decoding exhaustive search. So we have to very careful choosing the sphere radius.

In my work I am using VB (Viterbuo Boutros) sphere decoding algorithm and Single tree search sphere decoding algorithm for decoding. This algorithm use Maximum Likelihood (ML) decoding algorithm as basis. Here main objective is we try to locate the correct lattice point inside the sphere.

3. VB decoding algorithm

The direct approach to find the closest lattice point is to enumerate all lattice points falling inside a sphere centered at the received constellation point so as to identify the closest lattice point in the Euclidean metric. Basic reduction could be performed on the lattice generation matrix H to reduce the complexity of the decoding. In this case, Cholesky factorization is applied to the Gram matrix $G=HHT$ and it yields $G=WTW$, where W is an upper triangular matrix. The closest lattice point search problem is given as

$$\hat{u} = \arg \min_{u \in \Omega_{MT}} \|(p-x) W^2\|^2 \quad (4)$$

$$u \in \Omega_{MT}$$

Where vector $p=XH^{-1}$ is the least-mean-square (LMS) solution. The predefined parameter 'D' denotes the squared radius of an M-dimensional sphere centered at the receiving vector. So the closest lattice point must satisfy

M M

$$d(p,x) = \sum_{j=1}^M \sum_{i=j}^M W_{j,i} (p_j - x_j)^2 \leq D \quad (5)$$

j=1 i=j

Starting from the bottom row of matrix W and working backwards, the upper and lower bound of the examining lattice point can be determined from the above equation. We use x_k to represent the index of the examined layer and L_k to denote the upper bound of x_k . The closest lattice point search starts from the bottom layer to the top layer and monitor each lattice index from the lower region to the upper region. When the algorithm without violating the boundary constraint, reaches the highest layer a valid lattice point is found. Then the new distance d_{new} between the valid lattice point and the received vector signal is calculated and compared with the current best distance d_{best} . A closer lattice point to the received point is found if d_{new} is less than d_{best} . This lattice point is saved and the search radius is upgraded. The process iterates until all of the lattice points within the sphere are examined.

The procedures of the VB algorithm (sphere decoding algorithm) listed below:

- 1) **Preprocessing:** Transform H into an upper triangular matrix W by Cholesky decomposition algorithm. Initialize the sphere radius D by an adaptive method, set dimension index $k=N$ and $d_{best}=d$, and find the upper Bound L_k and index x_k .
- 2) **Finite-State Machine (FSM):** Upgrade $x_k = x_{k-1} + 1$ if ($x_k < L_k$ and $k > 1$), then go to State A; If ($x_k < L_k$ and $k=1$), then go to State B; If ($x_k \geq L_k$), then go to State C.
- 3) **State A:** Expand the search into $(k-1)$ sublayer, find the parameters used to upgrade x_k and L_k and, and go to State D.
- 4) **State B:** Compute d_{new} If $d_{new} < d_{best}$, record the currently best distance and the best lattice point, set $k=N$, and go to State D. If $d_{new} \geq d_{best}$, then go to FSM.
- 5) **State C:** If $k=N$, stop the algorithm. Otherwise, move the search one layer up $k=k+1$ and go to FSM.
- 6) **State D:** Upgrade x_k and L_k and that involve square root computations, and then go to FSM.

III. PROPOSED WORK

In botany, a tree is a plant with elongated stem, supporting branches and leaves. In data structure tree can be defined as collection of nodes (starts with root node), where each node is a data structure consist of values. In computer science, a search tree is a data structure to find required values from the set. In order for a tree to work as a search tree, the key of each node should be greater than keys in subtree of the left and less than keys of subtree in the right. The advantages of search tree is searching time is less if tree is balanced, balanced in tree mean number leaves in both sides are equal.

1. SINGLE TREE SEARCH SPHERE DECODING

In Decoding Tree associated with Decoding sphere the number of level in the tree is equal to Number of transmitting antenna in MIMO system. Each node in the tree will have as many children nodes as constellation size. In each level branches accumulated PED (partial Euclidean distance) higher than the sphere radius that path is discarded that reduce searching point very less compare with the sphere decoding. Hence performance of MIMO communication system is improved very much if Sphere decoding applied with the Tree search.

1. SINGLE TREE SEARCH ALGORITHM

The basic idea behind iterative tree-search algorithms lie in the conversion of the original ML decoding problem into a fully optimal ML decoding problem. ML detection is now changed into an equivalent tree search problem in which distances between received vector X and the received symbols Hs can be split into partial Euclidean distances (PEDs) $di(s(i))$ which depend only on $s(i)$ and which is a non-reducing non-negative function when proceeding from a parent node to its child node. Based on these PEDs, tree-search algorithms objective is finding the leaf node that is associated with the smallest $di(s(1))$ which is expected from the ML solution. To create a tree structure first QR factorization is applied on H matrix which converts it into a product of upper triangular matrix R and a unitary matrix Q .

$$WKT X=Hs+n$$

By applying QR factorization $H=QR$ then

$$X=QRs+n$$

Multiply both the sides by QH then $QHX=QH QRs+QHn$

Where $QH.Q=I$, then $QHX=Rs+QHn$

Assume $QHX=y$,

$$Y=Rs+QHn \quad (6)$$

2. Tree search using Sphere Decoding

Sphere decoding algorithm can be implemented by tree search algorithm. Key idea here is to reduce the number of lattice symbol vectors to be considered for the ML solution. Search includes only those symbol vectors S that lie inside an sphere of radius D .

$$\|x - Hs\|^2 < D^2 \quad (7)$$

This is known as sphere constraint (SC). Since it has been stated the Partial Euclidean distance (PEDs) are non-decreasing functions hence with reference to equation (7) algorithm visit only those nodes for which the following inequality holds:

$$di(s_i) < D^2 \quad (8)$$

If for any node equation (8) does not satisfy then one can prune the tree originating from that node without sustaining any performance penalty. Fig.2 shows how QR decomposition reduce the decoding complexity. An important aspect is how to select an initial radius d to start with. One way is to start with an initial value and dynamically update it as one finds a leaf node with smaller PED than Sphere constraint (SC). Choice of radius tell how fast the searching algorithm will produce the result. If the radii are large too many nodes will satisfy the SC and search will take more time. However, if the chosen radius is too small then no node may in the sphere and no solution will be found. Hence throughput of this algorithm is varied.

$$d(s) = \left\| \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} - \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} \right\|^2 \Leftrightarrow \left\| \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \\ \hat{y}_4 \end{bmatrix} - \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ 0 & R_{22} & R_{23} & R_{24} \\ 0 & 0 & R_{33} & R_{34} \\ 0 & 0 & 0 & R_{44} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} \right\|^2$$

Fig.2 Finding the distance before and after QR decomposition

3. Tree pruning

Tree pruning reduce the complexity of the Tree structure algorithm. The basic is to reduce the number of tree nodes visited to get a ML solution. In the tree structure which part of tree visited and which part of tree not visited (Tree pruning) based on the PED. Based on tree pruning strategy the algorithms achieve optimal BER performance. Figure 3 shows the effect of pruning with initial SC is set to 1. If leaf node with weight smaller than SC is found, SC is updated. Nodes with weight greater than current SC are then pruned (shown with dashed arrows).

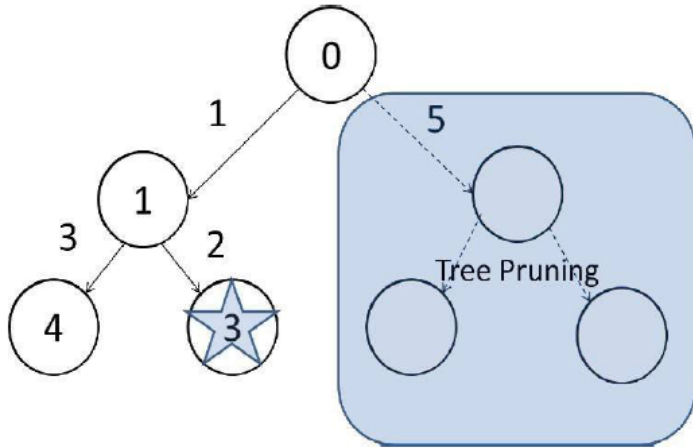


FIG3.TREE PRUNING

IV. RESULT AND DISCUSSION

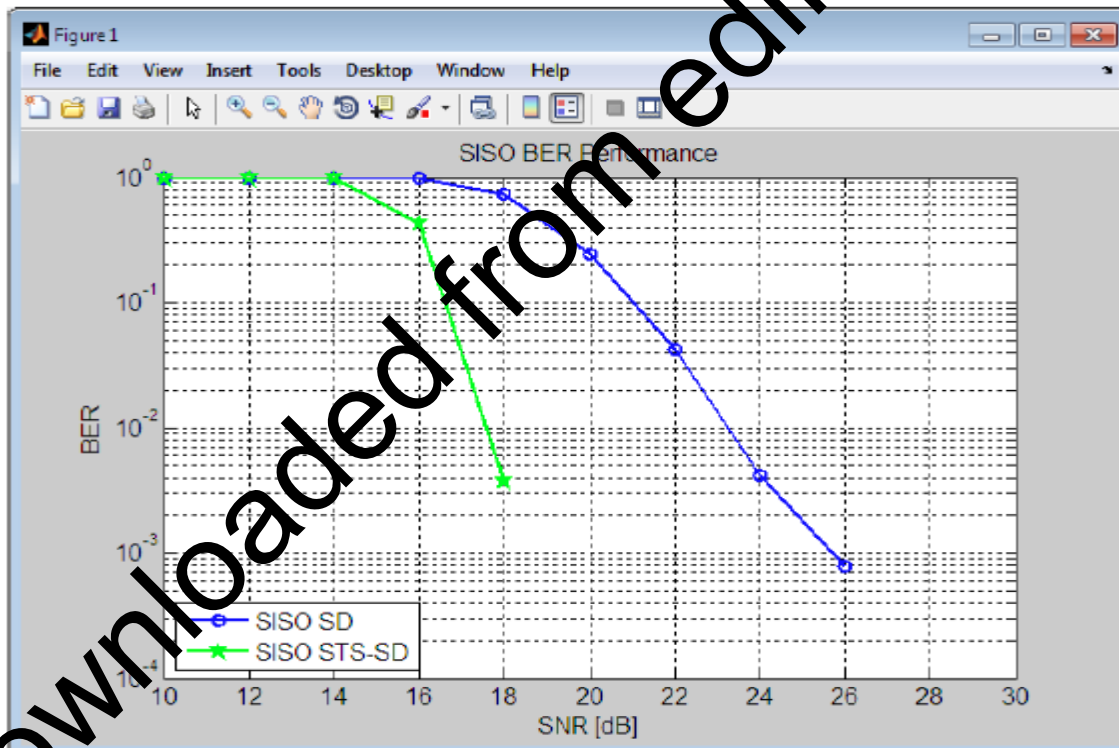


FIG 4 SNR VERSUS BER

SISO SD-SOFT INPUT SOFT OUTPUT SPHERE DECODING for 4*4 MIMO System with 16-QAM.

FIG 4 shows that the graph between SNR (Signal to Noise ratio) versus BER (Bit Error Rate). SISO STS-SD (Soft input Soft output Single Tree search- Sphere decoding) gives good BER performance compared with the SISO-SD (Soft input soft output-Sphere Decoding).

CONCLUSION

4*4 MIMO system with 16-QAM modulation is implemented using Matlab for SISO-SD and SISO STS-SD method. When apply the sphere decoding in tree structure with tree pruning concept, the complexity of MIMO Sphere decoding is reduced more. So that BER performance of STS-SD is improve lot compare with SISO-SD. In future same method we may try for Higher order constellation like 32-QAM, 64-QAM etc with 5*5 MIMO, 6*6 MIMO system etc. This algorithm may be tried to implement in Real Time FPGA and ASIC Back end to improve the Decoding rate, Reduce power consumption and Chip-area.

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